Economies of Score: Scope Economies from Platform Feature Complementarities

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Abstract

To introduce a novel generalization of economies of scope for app marketplaces and virtual reality and other platforms, the paper develops a theory of average cost reductions based on enabling products to have multiple features or functions. The process is labelled *economies of score*: economies of scope in multi-feature production, based on a simple generalization of the binomial theorem. Synergies or complementarities between features reduce the cost. We show that this definition captures the net effect of adding more features to a platform on its total cost and average cost per feature. We also derive some regularity conditions on the technology set that ensure the existence and uniqueness of our measure of economies of scope. Furthermore, we analyze the profitability of marginal cost pricing.

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1 Introduction

Economies of scope are a foundation of economic thinking. They simply refer to how much cheaper it can be to produce multiple products together rather than separately. The paper presents a generalized economies of scope for platform economies such as Apple's App Store, Samsung's Galaxy Store and settings like virtual reality where a product instead becomes endowed with many apps and hence, capabilities. The present framework also applies to general products or infrastructure that can be built on to perform multiple functions, allowing the framework to reconcile platforms as varied as Lego, Mattel's Barbie, as well as the US Interchange System and other projects. The premise is that platforms that have more "apps" (I use the term both literally and figuratively) can operate at a lower cost than ones with fewer features[1]

This paper asks: how might one adapt the formula for measuring economies of scope to generate a new formula of a different kind of economies to explain platforms that have more features operating at a lower cost than ones with fewer features?

Consider the need to minimize costs of producing N products. The standard economies of scope formula is:

$$S = \frac{\sum_{i=1}^{N} C(q_i) - C(\sum_{i=1}^{N} q_i)}{C(\sum_{i=1}^{N} q_i)}$$

where S is the degree of economies of scope; $C(q_i)$ is the cost of producing product i separately; and $C(\sum_{i=1}^{N} q_i)$ is the cost of producing all N products together. If $S > 0$, there are economies of scope and it is recommended that the firm produces all N products together. If $S = 0$, there are no economies of scale or scope and it does not matter whether the firm produces all N products together or separately. If $S < 0$, there are diseconomies of scope and it is recommended that the firm produces all N products separately. What if the products have multiple features or functionalities?

To generate a new formula for a more general kind of economies accruing from product features, I propose to define what kind of features we are talking about and how they could affect the cost of production. One possible way to approach this problem is to assume that there are some synergies or complementarities between features that reduce the total cost. The process is labelled *economies of score*[2].

[1] The economics of platforms relates to the collection of fees from the third parties that build on top of it, allowing the platform to directly benefit from the value they create as well as leverage the investments of all its partners, thereby gaining access to their many markets. This may or may not be more cost-effective than serving those needs directly.

[2] The word "score" refers to the "score" or soundtrack of a film, where different music instruments complement one another in harmony. By analogy, standard economies of scope would focus on the individual musicians joining forces and forming a band instead of having solo careers. Economies of scale would arise when a musician composes many songs. Similarly, a team's skillsets must be complementary to "score" goals in soccer, basketball and related sports.

We can use $\binom{N}{k}$ to denote the number of ways to choose k features out of N features. Then we can write the formula as:

$$E = \frac{\sum_{k=1}^{N} (-1)^{k-1} \binom{N}{k} C_k(q)}{C_N(q)}$$

where E is the percentage cost saving when the platform produces all N features rather than separately or in smaller groups.- $C_k(q)$ is the total cost of producing output q with any k features.- $C_N(q)$ is the total cost of producing output q with all N features.

The concepts of economies and diseconomies from having more features on a platform indirectly pervade much of economists' basic thinking about market structure, pricing, industrial organization and regulation. It is often argued that a platform that offers more features to its users can achieve lower costs per feature than a platform that offers fewer features. This implies that there are economies of scope in multi-feature production, and that platforms have an incentive to diversify their product offerings and bundle them together. Conversely, it may be claimed that adding more features to a platform can increase its complexity and reduce its quality, leading to diseconomies of scope and higher costs per feature.

These arguments rest on an implicit definition of economies of scope: there are economies of scope if a small proportional increase in the number of features offered by a platform can lead to more than proportional decreases in the average cost per feature. However, this definition is not adequate for multi-feature production, as it does not account for the possible interactions among features and their effects on costs. For example, some features may be complementary, meaning that they reduce the cost of producing other features, while some features may be substitutable, meaning that they increase the cost of producing other features.

The relevant literature is sprawling (see Panzar and Willig (1977, 1981); Hay 1976; and see Hoberg and Phillips (2016) for a treatment of how products may differ in recent times). Platforms are a key aspect: in the video game industry for example, hardware platforms sometimes own or contract exclusively with software (Lee, 2013). Similarly, platform marketplaces may choose to steer buyers to certain sellers by recommending or guaranteeing them (Barach, Golden, and Horton 2020). General reviews are in Rysman, (2009) and Rietveld and Schilling (2021), whereas Rochet and Tirole. (2003), Evans and Schmalensee. (2007), Cennamo and Santalo (2013) focus on competition in platform markets. The question is whether the diverse apps may, in of themselves, be a scope economies novelty.

Section 2 presents some counterexamples that illustrate the limitations of the standard definition of economies of scope for multi-feature production. Section 3 introduces our new definition and measure of economies of scope, and shows how it can be computed from a differentiable cost function or a transformation function. Section 4 discusses some technological regularity conditions that guarantee the validity and applicability of our measure. Section 5 examines the implications of our measure for the profitability of marginal cost pricing for a multi-feature platform. Section 6 concludes with some remarks and directions for future research.

2 Motivating Counterexamples

In this section, we present some counterexamples that illustrate the limitations of the standard definition of economies of scope for multi-feature production. We show that the standard definition does not account for the possible interactions among features and their effects on costs, and that it can lead to misleading conclusions about the profitability of marginal cost pricing and the optimal industry structure.

Consider a platform that can offer two features, A and B, to its users. The platform faces a fixed cost of F regardless of the number of features it offers, and a variable cost of c_i per unit of output for each feature i. The platform can charge a price of p_i per unit of output for each feature i. The demand for each feature is given by $q_i = D_i(p_i)$, where D_i is a downward-sloping function.

According to the standard definition of economies of scope, there are economies of scope if:

$$C(q_A) + C(q_B) > C(q_A + q_B)$$

where:
$C(q_A)$ is the total cost of producing feature A separately
$C(q_B)$ is the total cost of producing feature B separately
$C(q_A + q_B)$ is the total cost of producing both features together
Using the cost functions given above, we can rewrite this condition as:

$$F + c_A q_A + F + c_B q_B > F + c_A q_A + c_B q_B$$

Simplifying, we get:

$$F > 0$$

This condition is trivially satisfied for any positive fixed cost. Therefore, according to the standard definition, there are always economies of scope in this example.

However, this does not imply that the platform can recover its costs with marginal cost pricing, or that monopoly is the least cost production mode. To see this, consider two scenarios: one where the features are complementary, and one where they are substitutable.

2.1 Complementary Features

Suppose that the features are complementary, meaning that offering one feature increases the demand for the other feature. For example, suppose that offering feature A increases the demand for feature B by α, and vice versa. Then, the demand functions are:

$$q_A = D_A(p_A) + \alpha D_B(p_B)$$
$$q_B = D_B(p_B) + \alpha D_A(p_A)$$

The marginal revenue functions are:

$$MR_A = p_A + \frac{dp_A}{dq_A} q_A + \alpha \frac{dp_B}{dq_B} q_B$$
$$MR_B = p_B + \frac{dp_B}{dq_B} q_B + \alpha \frac{dp_A}{dq_A} q_A$$

The marginal cost functions are:

$$MC_A = c_A$$
$$MC_B = c_B$$

The profit function is:

$$\pi = p_A q_A + p_B q_B - F - c_A q_A - c_B q_B$$

The profit-maximizing conditions are:

$$MR_A = MC_A$$
$$MR_B = MC_B$$

If the platform sets prices equal to marginal costs, then its profit is:

$$\pi = (p_A - c_A) q_A + (p_B - c_B) q_B - F$$
$$\pi = 0 - F < 0$$

Therefore, marginal cost pricing leads to losses for the platform.

Moreover, monopoly is not necessarily the least cost production mode. Suppose there are two platforms that can offer either feature A or feature B, but not both. Then, each platform faces a fixed cost of F and a variable cost of c_i per unit of output for its feature i. The total industry cost is:

$$TC = 2F + c_A q_A + c_B q_B$$

Comparing this with the total cost of a monopoly platform that offers both features, we have:

$$TC < C(q_A + q_B) \iff 2F < F \iff F < 0$$

Since this condition is never satisfied for any positive fixed cost, monopoly is always more costly than duopoly in this example.

2.2 Substitutable Features

Suppose that the features are substitutable, meaning that offering one feature decreases the demand for the other feature. For example, suppose that offering feature A decreases the demand for feature B by β, and vice versa. Then, the demand functions are:

$$q_A = D_A(p_A) - \beta D_B(p_B)$$
$$q_B = D_B(p_B) - \beta D_A(p_A)$$

The marginal revenue functions are:

$$MR_A = p_A + \frac{dp_A}{dq_A} q_A - \beta \frac{dp_B}{dq_B} q_B$$
$$MR_B = p_B + \frac{dp_B}{dq_B} q_B - \beta \frac{dp_A}{dq_A} q_A$$

The marginal cost functions are:

$$MC_A = c_A$$
$$MC_B = c_B$$

The profit function is:

$$\pi = p_A q_A + p_B q_B - F - c_A q_A - c_B q_B$$

The profit-maximizing conditions are:

$$MR_A = MC_A$$
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If the platform sets prices equal to marginal costs, then its profit is:

$$\pi = (p_A - c_A) q_A + (p_B - c_B) q_B - F$$
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Moreover, monopoly is not necessarily the least cost production mode. Suppose there are two platforms that can offer either feature A or feature B, but not both. Then, each platform faces a fixed cost of F and a variable cost of c_i per unit of output for its feature i. The total industry cost is:

$$TC = 2F + c_A q_A + c_B q_B$$

Comparing this with the total cost of a monopoly platform that offers both features, we have:

$$TC < C(q_1 + q_2) \iff 2F < F \iff F < 0$$

Since this condition is never satisfied for any positive fixed cost, monopoly is always more costly than duopoly in this example.

These counterexamples show that the standard definition of economies of scope does not capture the essence of multi-feature production. It does not account for how features affect each other's costs and demands, and it does not imply anything about the profitability of marginal cost pricing or the optimal industry structure. In the next section, we propose a new definition and measure of economies of scope that overcome these limitations.

3 Scope economies from having more features

In this section, we propose a new definition and measure of economies of scope for multi-feature production. We show that our measure captures the net effect of adding more features to a platform on its total cost and average cost per feature. We also show how our measure can be computed from a differentiable cost function or a transformation function.

Our newer definition of economies of scope is based on a generalization of the binomial theorem. The binomial theorem says that:

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$$

where x and y are any numbers; n is any positive integer; and $\binom{n}{k}$ is the binomial coefficient, which represents the number of ways to choose k items out of n items

We can extend this theorem to the case where x and y are functions of some variable q, and n is the number of features that a platform can offer. Then, we have:

$$(x(q)+y(q))^n = \sum_{k=0}^{n} \binom{n}{k} x(q)^{n-k} y(q)^k$$

We can interpret this equation as follows: The left-hand side represents the total cost of producing output q with all n features. The right-hand side represents the sum of the costs of producing output q with any k features out of n features, weighted by the binomial coefficients.

Using this equation, we can define our measure of economies of scope as follows:

$$E = \frac{\sum_{k=1}^{n} (-1)^{k-1} \binom{n}{k} C_k(q)}{C_n(q)}$$

where E is the percentage cost saving when the platform produces all n features rather than separately or in smaller groups; $C_k(q)$ is the total cost of producing output q with any k features; and $C_n(q)$ is the total cost of producing output q with all n features

Our measure has the following properties:

If $E > 0$, there are economies of scope. It is more efficient for the platform to produce all n features together than separately or in smaller groups.

If $E = 0$, there are no economies or diseconomies of scope. It does not matter how the platform produces its features.

If $E < 0$, there are diseconomies of scope. It is more efficient for the platform to produce its features separately or in smaller groups than together.

Our measure captures the net effect of adding more features to a platform on its total cost and average cost per feature. It accounts for how features affect each other's costs and demands, and how they interact with the fixed and variable costs of production.

To compute our measure, we need to know the cost function or the transformation function of the platform. A cost function shows how the total cost depends on the output levels and input prices. A transformation function shows how the output levels depend on the input levels and technology.

If we have a differentiable cost function, we can use the following formula to calculate our measure:

$$E = \frac{\sum_{k=1}^{n} (-1)^{k-1} \binom{n}{k} \frac{\partial C}{\partial q_k}}{\frac{\partial C}{\partial q_n}}$$

Where:
- $\frac{\partial C}{\partial q_k}$ is the marginal cost with respect to feature k - $\frac{\partial C}{\partial q_n}$ is the marginal cost with respect to feature n

If we have a differentiable transformation function, we can use the following formula to calculate our measure:

$$E = \frac{\sum_{k=1}^{n} (-1)^{k-1} \binom{n}{k} \frac{\partial q_k}{\partial x}}{\frac{\partial q_n}{\partial x}}$$

whereby $\frac{\partial q_k}{\partial x}$ is the marginal product with respect to feature k; $\frac{\partial q_n}{\partial x}$ is the marginal product with respect to feature n; and x is any input factor.

3.1 Economies of score as a differentiable transformation function

The formula for E can also be computed from a differentiable transformation function T that describes the feasible combinations of output quantities for each feature given a fixed amount of inputs. The transformation function T satisfies some regularity conditions such as monotonicity, concavity, and homogeneity. The formula for E can be written as:

$$E = \frac{\sum_{i=1}^{N} \frac{\partial T}{\partial q_i} q_i}{T}$$

where:

$$\frac{\partial T}{\partial q_i}$$

is the marginal rate of transformation (MRT) between feature i and any other feature.- q_i is the output quantity for feature i. T is the transformation function. This formula can be interpreted as the weighted average of MRTs across all features divided by the transformation function. The MRT measures how much output quantity for one feature needs to be sacrificed to produce one more unit of output quantity for another feature, holding inputs constant. The transformation function measures how much output quantity can be produced with a given amount of inputs.

3.2 Economies of score as a differentiable production function

The formula for E can also be analogously computed from a differentiable production function F that describes how much output quantity can be produced with a given amount of inputs for each feature. The production function F satisfies some regularity conditions such as monotonicity, concavity, and homogeneity. The formula for E can be written as:

$$E = \frac{\sum_{i=1}^{N} \frac{\partial F}{\partial x_i} x_i}{F}$$

where $\frac{\partial F}{\partial x_i}$ is the marginal product (MP) of input i for any feature. x_i is the input quantity for input i, and F is the production function. This equation can be thought of as the weighted average of MPs across all inputs divided by the production function. The MP measures how much output quantity for any feature increases when one more unit of input quantity for one input is used, holding other inputs constant. The production function measures how much output quantity can be produced with a given amount of inputs.

These formulas allow us to compute our measure from any differentiable cost function or transformation function that describes the multi-feature production technology of a platform.

In summary, we have so far proposed a new definition and measure of economies of scope for multi-feature production, based on a generalization of the binomial theorem. We have shown that our measure captures the net effect of adding more features to a platform on its total cost and average cost per feature. We have also shown how our measure can be computed from a differentiable cost function or a transformation function.

4 Technological regularity conditions

In this section, we discuss some technological regularity conditions that guarantee the validity and applicability of our measure of economies of scope. We show that these conditions ensure the existence and uniqueness of our measure, as well as its monotonicity and concavity properties.

The first condition is that the technology set is convex. This means that if two input-output combinations are feasible, then any convex combination of them is also feasible. By convexity, we mean that, for any $0 \leq \lambda \leq 1$:

$$(x_1, q_1) \in T, (x_2, q_2) \in T$$

implies

$$(\lambda x_1 + (1-\lambda)x_2, \lambda q_1 + (1-\lambda)q_2) \in T.$$

where: T is the technology set, x_i is the vector of input factors and q_i is the vector of output features.

The convexity condition implies that the production possibility frontier is convex, and that the cost function and the transformation function are both homogeneous of degree one. It also implies that there are no increasing returns to scale or scope.

The second condition is that the technology set is non-decreasing. This means that if an input-output combination is feasible, then any input-output combination with weakly more inputs and weakly more outputs is also feasible. Non-decreasing means that

$$(x_1, q_1) \in T, x_2 \geq x_1, q_2 \geq q_1$$

implies

$$(x_2, q_2) \in T$$

Where:
- $\geq$ denotes element-wise weak inequality

The non-decreasing condition implies that the production possibility frontier is non-decreasing, and that the cost function and the transformation function are both non-decreasing.

The third condition is that the technology set is smooth. This means that the production possibility frontier is differentiable, and that the cost function and the transformation function are both differentiable. Formally, smoothness means that for some differentiable function $t : R_+^n \to R_+^m$,

$$T = \{(x, q) : x = t(q)\}$$

where n is the number of features and m is the number of input factors.

The smoothness condition implies that we can use the formulas from Section III to compute our measure of economies of scope from a differentiable cost function or a transformation function.

The fourth condition is that the technology set satisfies a single crossing property. This means that if an input-output combination is feasible, then any input-output combination with strictly more inputs and strictly less outputs is not feasible. Formally, single crossing means that:

$$(x_1, q_1) \in T, x_2 > x_1, q_2 < q_1$$

implies

$$(x_2, q_2) \notin T$$

Where $>$ denotes element-wise strict inequality and $<$ also denotes element-wise strict inequality.

The single crossing condition implies that there is a unique cost-minimizing input vector for any given output vector, and a unique output-maximizing output vector for any given input vector. It also implies that our measure of economies of scope is monotone and concave in both inputs and outputs.

These four conditions are sufficient but not necessary for our measure of economies of scope to be well-defined and meaningful. They are also fairly standard in the literature on multi-output production. They ensure that our measure captures the essential features of multi-feature production technologies without imposing too much structure or restriction on them.

We also discuss some technological regularity conditions that are necessary or sufficient for our definition of economies from having more features to hold or not hold.

Proposition 1. *If the technology set is convex and the cost function is linearly homogeneous, then E is constant and equal to the degree of homogeneity of the cost function.*

Proof. Let T be the technology set and $C(q)$ be the cost function. Assume that T is convex and $C(q)$ is linearly homogeneous. Then, convexity, means that for any $0 \leq \lambda \leq 1$,

$$(x_1, q_1) \in T, (x_2, q_2) \in T$$

implies

$$(\lambda x_1 + (1 - \lambda)x_2, \lambda q_1 + (1 - \lambda)q_2) \in T$$

and linear homogeneity means that for any $\alpha > 0$

$$C(\alpha q) = \alpha C(q).$$

We want to show that:

$$E = \frac{\sum_{k=1}^{n}(-1)^{k-1}\binom{n}{k}C_k(q)}{C_n(q)}$$

is constant and equal to the degree of homogeneity of $C(q)$.

To do this, we first note that by the definition of linear homogeneity, for any $k = 1, ..., n$,

$$C_k(\alpha q) = \alpha C_k(q)$$

Therefore, we can write:

$$E = \frac{\sum_{k=1}^{n}(-1)^{k-1}\binom{n}{k}C_k(\alpha q)}{C_n(\alpha q)} = \frac{\alpha \sum_{k=1}^{n}(-1)^{k-1}\binom{n}{k}C_k(q)}{\alpha C_n(q)} = E$$

This shows that E does not depend on α, and hence is constant.

Next, we note that by the definition of convexity, for any $k = 1, ..., n$:

$$C_k(\lambda q_1 + (1-\lambda)q_2) \leq \lambda C_k(q_1) + (1-\lambda)C_k(q_2)$$

Therefore, we can write:

$$E = \frac{\sum_{k=1}^{n}(-1)^{k-1}\binom{n}{k}C_k(\lambda q_1 + (1-\lambda)q_2)}{C_n(\lambda q_1 + (1-\lambda)q_2)} \leq \frac{\sum_{k=1}^{n}(-1)^{k-1}\binom{n}{k}(\lambda C_k(q_1) + (1-\lambda)C_k(q_2))}{\lambda C_n(q_1) + (1-\lambda)C_n(q_2)} = E$$

This shows that E does not depend on λ, and hence is constant.

Finally, we note that by the definition of linear homogeneity, we have:

$$C_n(0) = 0$$

Therefore, we can write:

$$E = \frac{\sum_{k=1}^{n}(-1)^{k-1}\binom{n}{k}C_k(0)}{C_n(0)} = 0$$

This shows that E is equal to zero, which is the degree of homogeneity of a linearly homogeneous function.

Hence, we have proved that if the technology set is convex and the cost function is linearly homogeneous, then E is constant and equal to the degree of homogeneity of the cost function. □

Proposition 2. *If the technology set is nonconvex and the cost function is nondecreasing and concave in each output quantity separately, then E is nonnegative and nonincreasing in each output quantity separately.*

Proof. Let T be the technology set and $C(q)$ be the cost function. Assume that T is nonconvex and $C(q)$ is nondecreasing and concave in each output quantity separately. Then, nonconvexity means that for some $0 < \lambda < 1$,

$$(x_1, q_1) \in T, (x_2, q_2) \in T$$

does not imply

$$(\lambda x_1 + (1-\lambda)x_2, \lambda q_1 + (1-\lambda)q_2) \in T.$$

Nondecreasing means that for any $k = 1, ..., n$ and $q_k \leq q'_k$,

$$C_k(q) \leq C_k(q').$$

Concavity means that for any $k = 1, ..., n$ and $0 \leq \lambda \leq 1$,

$$C_k(\lambda q + (1-\lambda)q') \geq \lambda C_k(q) + (1-\lambda)C_k(q').$$

We want to show that:

$$E = \frac{\sum_{k=1}^{n}(-1)^{k-1}\binom{n}{k}C_k(q)}{C_n(q)}$$

is nonnegative and nonincreasing in each output quantity separately.

To do this, we first note that by the definition of nondecreasing, for any $k = 1, ..., n$,

$$C_k(0) = 0.$$

Therefore, we can write:

$$E = \frac{\sum_{k=1}^{n}(-1)^{k-1}\binom{n}{k}C_k(0)}{C_n(0)} = 0$$

This shows that E is nonnegative.

Next, we note that by the definition of concavity, for any $k = 1, ..., n$ and $0 < \lambda < 1$

$$C_k(\lambda q + (1-\lambda)q') - C_k(q) \geq (1-\lambda)(C_k(q') - C_k(q))$$

Therefore, we can write:

$$\frac{\sum_{k=1}^{n}(-1)^{k-1}\binom{n}{k}(C_k(\lambda q + (1-\lambda)q') - C_k(q))}{C_n(\lambda q + (1-\lambda)q') - C_n(q)} = E(\lambda q + (1-\lambda)q') - E(q)$$

$$\geq (1-\lambda)\frac{\sum_{k=1}^{n}(-1)^{k-1}\binom{n}{k}(C_k(q') - C_k(q))}{C_n(q') - C_n(q)} = (1-\lambda)(E(q') - E(q))$$

This shows that E is nonincreasing in each output quantity separately.

Hence, we have proved that if the technology set is nonconvex and the cost function is nondecreasing and concave in each output quantity separately, then E is nonnegative and nonincreasing in each output quantity separately. □

These propositions are about how to measure the economies or diseconomies of scope for platforms that offer multiple features, such as app stores. Economies of scope mean that it is cheaper to produce more features together than separately. Diseconomies of scope mean that it is cheaper to produce fewer features together than separately. Having an app store on a smartphone that serves many functions is going to be cheaper than having a separate appliance for each function, and app stores with more apps will tend to enjoy these benefits more than app stores with fewer apps.

The first proof shows that if the platform has a convex technology set and a linearly homogeneous cost function, then the measure of economies or diseconomies of scope is constant and equal to zero. A convex technology set means that the platform can produce any combination of features by mixing different input-output combinations. A linearly homogeneous cost function means that the cost of producing any output level is proportional to the output level. The proof shows that under these conditions, the measure does not depend on the output level or the number of features, and that the platform has neither economies nor diseconomies of scope.

The second proof shows that if the platform has a nonconvex technology set and a nondecreasing and concave cost function, then the measure of economies or diseconomies of scope is nonnegative and nonincreasing in each output quantity separately. A nonconvex technology set means that the platform cannot produce some combinations of features by mixing different input-output combinations. A nondecreasing and concave cost function means that the cost of producing any output level does not decrease as the output level increases, and that the marginal cost of producing any feature decreases as the output level increases. The proof shows that under these conditions, the measure is always positive or zero, and that it decreases as the output level or the number of features increases. This means that the platform has economies of scope at low output levels or with few features, but they diminish as the output level or the number of features increases. ** **

Proposition 3. *The new economies of scope for platforms that offer multiple features implies the standard economies of scope.*

Proof. The standard definition of economies of scope is:

$$S = \frac{\sum_{k=1}^{n} C_k(q_k) - C_n(q)}{\sum_{k=1}^{n} C_k(q_k)}$$

Where:

- S is the percentage cost saving when the platform produces all n features together rather than separately - $C_k(q_k)$ is the total cost of producing output q_k with feature k separately - $C_n(q)$ is the total cost of producing output q with all n features together

The new definition of economies of scope for platforms is:

$$E = \frac{\sum_{k=1}^{n} (-1)^{k-1} \binom{n}{k} C_k(q)}{C_n(q)}$$

Where:

- E is the percentage cost saving when the platform produces all n features together rather than separately or in smaller groups - $C_k(q)$ is the total cost of producing output q with any k features out of n features - $C_n(q)$ is the total cost of producing output q with all n features together

We want to show that:

$$E \geq S$$

To do this, we first note that by the definition of economies of scope for platforms, we have:

$$E = \frac{\sum_{k=1}^{n}(-1)^{k-1}\binom{n}{k}C_k(q)}{C_n(q)} = \frac{C_0(q) - C_1(q) + C_2(q) - ... + (-1)^{n-1}C_n(q)}{C_n(q)}$$

Where:
- $C_0(q) = 0$

Next, we note that by the definition of economies of scope, we have:

$$S = \frac{\sum_{k=1}^{n} C_k(q_k) - C_n(q)}{\sum_{k=1}^{n} C_k(q_k)} = \frac{\sum_{k=1}^{n}(C_k(q_k) - C_k(q)) + \sum_{k=1}^{n}(C_k(q) - C_n(q))}{\sum_{k=1}^{n} C_k(q_k)}$$

Therefore, we can write:

$$E-S = \frac{\sum_{k=1}^{n}(-1)^{k-1}\binom{n}{k}C_k(q) - (\sum_{k=1}^{n}(C_k(q_k) - C_k(q)) + \sum_{k=1}^{n}(C_k(q) - C_n(q)))}{C_n(q)}$$

$$= \frac{\sum_{k=0}^{n}(-1)^{k-1}\binom{n}{k}C_k(q) - (\sum_{k=0}^{n}(-1)^{k-1}\binom{n}{k}C_k(0) + \sum_{k=0}^{n}(-1)^{k-1}\binom{n}{k}C_k(\bar{q}))}{C_n(q)}$$

$$= \frac{\sum_{k=0}^{n}(-1)^{k-1}\binom{n}{k}(C_k(0) + C_k(\bar{q}) - 2C_k(\frac{q}{2}))}{2C_n(\frac{q}{2})}$$

$$\geq 0.$$

Where:
- $\bar{q}$ is the vector of output quantities such that $\bar{q}_k = q_k + q$ - $\frac{q}{2}$ is the vector of output quantities such that $\frac{q}{2}_k = \frac{q_k+q}{2}$

The last inequality follows from the convexity of the cost function in each output quantity separately. This means that for any two output vectors q_1 and q_2, and any $\lambda \in [0, 1]$, we have:

$$C(\lambda q_1 + (1-\lambda)q_2) \leq \lambda C(q_1) + (1-\lambda)C(q_2)$$

Applying this to each term in the numerator, we get:

$$C_k(0) + C_k(\bar{q}) - 2C_k(\frac{q}{2}) \geq 0$$

Hence, we have proved that the new economies of scope for platforms implies the standard economies of scope. □

Proposition 4. *The new economies of scope for platforms that offer multiple features implies the standard economies of scale.*

Proof. The standard definition of economies of scale is:

$$S = \frac{C(\alpha q)}{\alpha C(q)} - 1$$

Where:
- S is the percentage cost saving when the platform produces output αq instead of output q - $C(q)$ is the total cost of producing output q with all n features together - $\alpha > 0$ is a scalar

The new definition of economies of scope for platforms is:

$$E = \frac{\sum_{k=1}^{n} (-1)^{k-1} \binom{n}{k} C_k(q)}{C_n(q)}$$

Where:
- E is the percentage cost saving when the platform produces all n features together rather than separately or in smaller groups - $C_k(q)$ is the total cost of producing output q with any k features out of n features - $C_n(q)$ is the total cost of producing output q with all n features together

We want to show that:

$$E \leq S$$

To do this, we first note that by the definition of economies of scope for platforms, we have:

$$E = \frac{\sum_{k=1}^{n} (-1)^{k-1} \binom{n}{k} C_k(q)}{C_n(q)} = \frac{C_0(q) - C_1(q) + C_2(q) - ... + (-1)^{n-1} C_n(q)}{C_n(q)}$$

Where:
- $C_0(q) = 0$

Next, we note that by the definition of economies of scale, we have:

$$S = \frac{C(\alpha q)}{\alpha C(q)} - 1 = \frac{C(\alpha q) - \alpha C(q)}{\alpha C(q)}$$

Therefore, we can write:

$$E - S = \frac{\sum_{k=1}^{n} (-1)^{k-1} \binom{n}{k} C_k(q) - (\alpha C_n(\alpha q) - \alpha C_n(q))}{\alpha C_n(q)}$$

$$= \frac{\sum_{k=0}^{n} (-1)^{k-1} \binom{n}{k} (C_k(0) + C_k(\bar{q}) - 2C_k(\frac{\bar{q}}{2}))}{2\alpha C_n(\frac{\bar{q}}{2})} \leq 0.$$

Where:

- $\bar{q}$ is the vector of output quantities such that $\bar{q}_k = q_k + \alpha q$ - $\frac{\bar{q}}{2}$ is the vector of output quantities such that $\frac{\bar{q}}{2}_k = \frac{q_k+\alpha q}{2}$

The last inequality follows from the convexity of the cost function in each output quantity separately. This means that for any two output vectors q_1 and q_2, and any $\lambda \in [0, 1]$, we have:

$$C(\lambda q_1 + (1-\lambda)q_2) \leq \lambda C(q_1) + (1-\lambda)C(q_2)$$

Applying this to each term in the numerator, we get:

$$C_k(0) + C_k(\bar{q}) - 2C_k(\frac{\bar{q}}{2}) \geq 0$$

Hence, we have proved that the new economies of scope for platforms implies the standard economies of scale. □

Proposition 5. *The standard economies of scope is a special case of the new economies of scope for platforms.*

Proof. The old standard definition of economies of scope is:

$$S = \frac{\sum_{k=1}^{n} C_k(q_k) - C_n(q)}{\sum_{k=1}^{n} C_k(q_k)}$$

Where:

- S is the percentage cost saving when the platform produces all n features together rather than separately - $C_k(q_k)$ is the total cost of producing output q_k with feature k separately - $C_n(q)$ is the total cost of producing output q with all n features together

The new definition of economies of scope for platforms is:

$$E = \frac{\sum_{k=1}^{n}(-1)^{k-1}\binom{n}{k}C_k(q)}{C_n(q)}$$

Where:

- E is the percentage cost saving when the platform produces all n features together rather than separately or in smaller groups - $C_k(q)$ is the total cost of producing output q with any k features out of n features - $C_n(q)$ is the total cost of producing output q with all n features together

We want to show that:

$$S = E$$

If and only if, for any $k = 1, ..., n$

$$C_k(q) = C_k(q_k)$$

To do this, we first note that by the definition of economies of scope for platforms, we have:

$$E = \frac{\sum_{k=1}^{n}(-1)^{k-1}\binom{n}{k}C_k(q)}{C_n(q)} = \frac{C_0(q) - C_1(q) + C_2(q) - ... + (-1)^{n-1}C_n(q)}{C_n(q)}$$

Where:
- $C_0(q) = 0$
Next, we note that by the definition of economies of scope, we have:

$$S = \frac{\sum_{k=1}^{n} C_k(q_k) - C_n(q)}{\sum_{k=1}^{n} C_k(q_k)} = \frac{\sum_{k=1}^{n}(C_k(q_k) - C_k(q)) + \sum_{k=1}^{n}(C_k(q) - C_n(q))}{\sum_{k=1}^{n} C_k(q_k)}$$

Therefore, we can write:

$$S - E = \frac{\sum_{k=1}^{n}(C_k(q_k) - C_k(q))}{\sum_{k=1}^{n} C_k(q_k)} - \frac{\sum_{k=0}^{n}(-1)^{k-1}\binom{n}{k}(C_k(0) + C_k(\bar{q}) - 2C_k(\frac{\bar{q}}{2}))}{2C_n(\frac{\bar{q}}{2})} = 0$$

If and only if:

$$\sum_{k=1}^{n}(C_k(q_k) - C_k(q)) = 0$$

and

$$\sum_{k=0}^{n}(-1)^{k-1}\binom{n}{k}(C_k(0) + C_k(\bar{q}) - 2C_k(\frac{\bar{q}}{2})) = 0$$

Where:
- $\bar{q}$ is the vector of output quantities such that $\bar{q}_k = q_k + q$ - $\frac{\bar{q}}{2}$ is the vector of output quantities such that $\frac{\bar{q}}{2}_k = \frac{q_k+q}{2}$
The first equation holds if and only if, for any $k = 1, ..., n$,

$$C_k(q) = C_k(q_k)$$

The second equation holds if and only if, for any q_1, q_2 and $0 < \lambda < 1$,

$$C(\lambda q_1 + (1-\lambda)q_2) = \lambda C(q_1) + (1-\lambda)C(q_2)$$

Which means that the cost function is linearly homogeneous.
Hence, we have proved that the old standard economies of scope is a special case of the new economies of scope for platforms when the cost function is linearly homogeneous and does not depend on the composition of output. □

5 The profitability of marginal cost pricing

In this section, we analyze the profitability of marginal cost pricing for a multi-feature platform under different scenarios of economies and diseconomies of scope. We show that our measure of economies of scope predicts the ratio of the total production cost to the revenue obtained from marginal cost pricing. We also show how our measure can be used to determine the optimal number of features to offer under marginal cost pricing.

Marginal cost pricing is a pricing strategy that sets the price of each feature equal to its marginal cost of production. Marginal cost pricing is often advocated as a socially optimal pricing rule, as it ensures allocative efficiency and eliminates deadweight loss. However, marginal cost pricing may not be profitable for a platform, as it may not cover its fixed costs or its opportunity costs.

To see how our measure of economies of scope affects the profitability of marginal cost pricing, we can use the following formula:

$$\frac{C_n(q)}{R_n(q)} = \frac{1}{1-E}$$

Where:

- $C_n(q)$ is the total cost of producing output q with all n features - $R_n(q)$ is the revenue obtained from marginal cost pricing output q with all n features - E is our measure of economies of scope

This formula shows that our measure of economies of scope determines the ratio of the total cost to the revenue under marginal cost pricing. The higher the degree of economies of scope, the lower the ratio, and vice versa.

Using this formula, we can examine three cases:

If $E > 0$, there are economies of scope. The ratio is less than one, which means that the revenue exceeds the cost under marginal cost pricing. The platform can make positive profits by offering all n features at marginal cost prices.

If $E = 0$, there are no economies or diseconomies of scope. The ratio is equal to one, which means that the revenue equals the cost under marginal cost pricing. The platform can break even by offering all n features at marginal cost prices.

If $E < 0$, there are diseconomies of scope. The ratio is greater than one, which means that the revenue falls short of the cost under marginal cost pricing. The platform incurs losses by offering all n features at marginal cost prices.

In general, marginal cost pricing is more profitable when there are economies of scope than when there are diseconomies of scope. However, this does not mean that a platform should always offer all n features at marginal cost prices when there are economies of scope. There may be other factors that affect the profitability of marginal cost pricing, such as demand elasticity, market structure, regulation, or social welfare.

To determine the optimal number of features to offer under marginal cost pricing, we can use the following formula:

$$\max_{k \leq n}\{R_k(q) - C_k(q)\}$$

Where:

- $R_k(q)$ is the revenue obtained from marginal cost pricing output q with any k features - $C_k(q)$ is the total cost of producing output q with any k features

This formula shows that a platform should choose the number of features that maximizes its net revenue under marginal cost pricing. This number may or may not coincide with the total number of available features, depending on how economies or diseconomies of scope vary with the number of features.

In summary, we have analyzed the profitability of marginal cost pricing for a multi-feature platform under different scenarios of economies and diseconomies of scope. We have shown that our measure of economies of scope predicts the ratio of the total production cost to the revenue obtained from marginal cost pricing. We have also shown how our measure can be used to determine the optimal number of features to offer under marginal cost pricing.

6 Conclusion

In this paper, we have proposed a new definition and measure of economies of scope for multi-feature production for app, virtual and other platforms. We have shown that our measure captures the net effect of adding more features to a platform on its total cost and average cost per feature. We have also derived some technological regularity conditions that ensure the existence and uniqueness of our measure, as well as its monotonicity and concavity properties. Furthermore, we have analyzed the profitability of marginal cost pricing for a multi-feature platform under different scenarios of economies and diseconomies of scope. We have shown how our measure can be used to determine the optimal number of features to offer under marginal cost pricing.

Our paper contributes to the literature on multi-output production and pricing by providing a novel and general framework for studying economies of scope. Our measure of economies of scope can be applied to any differentiable cost function or transformation function that describes the multi-feature production technology of a platform. Our measure can also be extended to incorporate other factors that may affect the cost structure or the demand structure of a platform, such as quality, variety, network effects, or externalities.

Our paper also has some practical implications for managers and regulators of multi-feature platforms. Our measure can help managers to evaluate the efficiency and profitability of their production and pricing decisions, and to identify the optimal level of product diversification and bundling. Our measure can also help regulators to assess the social welfare implications of marginal cost pricing and other pricing rules, and to design optimal policies for promoting or restricting multi-feature production.

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