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Bio-Inspired Economics for Large Language Models: Optimizing Safety and Efficiency

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Abstract

Large Language Models (LLMs) have transformed natural language processing, yet their internal mechanisms remain opaque, posing challenges for interpretability and optimization. Recent work has applied biological analogies to dissect LLM reasoning through computational graphs, revealing how circuits contribute to behaviors like hallucinations. However, these insights lack a framework to evaluate their practical and societal implications. In this paper, we propose an economic lens to complement this biological approach, unpacking the economics of large language models in the process. We develop a novel methodology to analyze LLMs as economic systems, where circuits compete for computational resources (e.g., attention, activations) under constraints, akin to agents in a marketplace. To illustrate our approach, we present a case study with a hypothetical 1-billion-parameter LLM, EcoNet-1B, quantifying the costs and benefits of reasoning pathways identified in prior work. Using cost-benefit analysis and Value-at-Risk, we assess failure modes like hallucinations, showing that fine-tuning a "factual recall" circuit reduces hallucination rates by 15% (from 20% to 17%) at a compute cost of 5% of the original training ($500), yielding annual savings of $1.095 million in a customer support application. We further model the LLM as an ecosystem, where circuits "trade" influence to shape outputs, identifying "market failures" that lead to errors and offering economically grounded strategies to mitigate them. By bridging biology and economics, this work provides a unified bioeconomic framework to prioritize interpretability efforts, reduce societal risks, and guide sustainable LLM development. Our approach not only enhances the mechanistic understanding of LLMs but also makes their internal dynamics more actionable for engineers and policymakers, fostering broader engagement with interpretability research.

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1 Introduction

Large Language Models (LLMs) have revolutionized natural language processing, enabling unprecedented capabilities in tasks ranging from text generation to question answering and decision support (Brown et al., 2020; Raffel et al., 2020). As these models scale in size and complexity—reaching billions or even trillions of parameters—their internal mechanisms have become increasingly opaque, often described as "black boxes" (Rudin, 2019). This lack of transparency poses significant challenges for reliability, safety, and societal impact, particularly as LLMs are deployed in high-stakes domains such as healthcare, finance, and education (Bommasani et al., 2021). For instance, LLMs frequently exhibit hallucinations—generating factually incorrect or unsupported outputs—leading to risks like misinformation or costly errors in real-world applications (Ji et al., 2023; Visual Capitalist, 2025). Understanding and mitigating these failure modes requires moving beyond surface-level performance metrics to a deeper mechanistic understanding of how LLMs reason and make decisions.

Recent advances in interpretability have begun to address this challenge by dissecting the internal workings of LLMs. A landmark contribution in this space is Lindsey et al., (2025), which applies a biological analogy to study LLMs as complex organisms. Using novel techniques like circuit tracing, the authors map computational graphs within Claude 3.5 Haiku, revealing how distinct circuits handle multi-step reasoning and contribute to behaviors such as hallucinations. For example, they identify "known answer" circuits that misfire when the model recognizes a name but lacks sufficient details, leading to incorrect outputs. This biological framework provides a powerful lens for understanding LLM reasoning, offering insights into the modular structures that underpin their capabilities. However, while this approach excels at describing how LLMs function internally, it does not fully address the practical implications of these mechanisms—such as the computational costs of maintaining certain circuits, the economic risks of failure modes, or the trade-offs involved in optimizing model behavior.

In this paper, we propose a complementary perspective: an economic framework for analyzing LLMs. We introduce the economics of large language models which treats LLMs

as economic systems where circuits compete for computational resources (e.g., attention, activations) under constraints, akin to agents in a marketplace. This lens allows us to quantify the costs (e.g., energy, compute time, and other) and benefits (e.g., accuracy, robustness, risk reduction) of LLM reasoning pathways, providing a practical framework to prioritize interpretability efforts and guide model optimization.

Our approach builds on biological insights in the literature by asking not just how circuits operate, but what they cost and what value they provide—both to the model's performance and to its societal applications. For instance, while Lindsey et al., (2025) identifies a circuit responsible for hallucinations, we aim to evaluate the economic cost of such errors in a customer support context and the return on investment of mitigating them through fine-tuning. The need for an economic perspective is underscored by the growing complexity and societal stakes of LLMs. As models scale, their training and inference costs soar—GPT-3's training alone emitted 500 metric tons of CO2 (Lajavaness, 2024)—raising concerns about sustainability and accessibility (Trabelsi, 2023).

Simultaneously, the economic impact of LLM failures is becoming more pronounced, with hallucinations potentially costing businesses millions in high-stakes applications (Gu et al., 2020). Despite these challenges, the AI research community has been slow to engage with interpretability studies, as noted in recent discussions (austinc3301, 2025). By framing LLM behavior in economic terms, we aim to make interpretability more actionable and relevant, bridging the gap between mechanistic understanding and practical deployment. To illustrate our approach, we present a case study with a hypothetical 1-billion-parameter LLM, EcoNet-1B, designed to reflect the characteristics of lightweight models. We apply our economic framework to analyze and optimize a "factual recall" circuit responsible for hallucinations, demonstrating how fine-tuning can reduce hallucination rates by 15% at a compute cost of 5% of the original training, yielding significant economic benefits in a customer support application. This case study serves as a proof of concept, showing how economic analysis can complement biological insights to enhance LLM reliability and sustainability.

The remainder of this paper is structured as follows. Section 2 reviews related work on

LLM interpretability and economic analyses of AI systems, positioning our contribution at the intersection of these fields. Section 3 presents our economic framework, detailing the methodology for modeling LLMs as economic systems and quantifying circuit-level costs and benefits. Section 4 describes the EcoNet-1B case study, applying our framework to optimize model behavior and assess its economic impact. Section 5 discusses the implications of our findings for LLM development, safety, and societal deployment, while Section 6 concludes with directions for future research. By integrating biological and economic perspectives, we aim to provide a unified bioeconomic framework that not only deepens our understanding of LLMs but also makes their internal dynamics more actionable for engineers, policymakers, and researchers seeking to build more reliable and sustainable AI systems.

2 Related Work

2.1 Interpretability in Large Language Models

The rapid advancement of Large Language Models (LLMs) has spurred significant interest in understanding their internal mechanisms, a field broadly termed interpretability. Early interpretability efforts focused on post-hoc explanations, such as attention visualization (Vaswani et al., 2017) and feature importance scores (Ribeiro et al., 2016), which provide insights into model decisions but often fail to capture the underlying reasoning processes. More recently, the field of mechanistic interpretability has emerged, aiming to reverse-engineer LLMs by mapping their computational graphs and identifying modular structures, or "circuits," that handle specific tasks (Olah et al., 2020; Elhage et al., 2021).

A seminal contribution in this space is Lindsey et al., (2025), which adopts a biological analogy to study LLMs as complex organisms. Using a technique called circuit tracing, the authors dissect Claude 3.5 Haiku, a lightweight production model, to reveal how its reasoning unfolds across multiple steps. For example, they identify language-specific and language-independent circuits, showing that the latter dominate in larger models like Claude 3.5 Haiku, and pinpoint failure modes such as hallucinations caused by misfiring

"known answer" circuits (Lindsey et al, 2025). This biological framework provides a rich, human-understandable description of LLM behavior, enabling researchers to diagnose issues like factual inaccuracies in a manner akin to medical diagnostics. However, the approach is primarily descriptive, focusing on how circuits operate rather than evaluating their practical implications—such as the computational resources they consume or the economic risks they pose in real-world applications.

Other mechanistic interpretability studies have similarly advanced our understanding of LLMs. Elhage et al. (2021) identified induction heads in transformer models, circuits responsible for pattern-matchinging tasks, while Bricken et al. (2023) explored sparse coding techniques to extract interpretable features from LLMs. Despite these advances, challenges remain. Manual circuit tracing is labor-intensive and struggles to generalize across prompts, limiting its scalability. Our work seeks to address such gaps by introducing an economic lens that makes interpretability more actionable, quantifying the costs and benefits of circuits to prioritize optimization efforts.

2.2 Economic Analyses of AI Systems

Parallel to interpretability research, a growing body of work has examined the economics of AI systems, focusing on their computational costs, societal impacts, and risk profiles. The computational cost of training and deploying LLMs has been a major concern, particularly as models scale. Strubell et al. (2019) estimated that training a single transformer model can emit as much CO2 as five cars over their lifetimes, a finding echoed by Lajavaness (2024), which notes that GPT-3's training alone produced 500 metric tons of CO2. These costs extend beyond training to inference, with idle models in deployment adding to the economic and environmental burden (Li, 2023). Such analyses highlight the need for sustainable AI development, a priority also emphasized in the bioeconomy literature, which advocates for leveraging technology to mitigate environmental impacts (World Economic Forum, 2025).

Economic studies have also explored the societal impacts of AI. Acemoglu and Restrepo (2016) examined how AI-driven automation affects labor markets, identifying risks

like job polarization and inequality, while the Bank for International Settlements (BIS, 2024) investigated LLMs' applications in economics, such as forecasting and policy analysis, noting the potential for fine-tuning to improve performance in specific domains. However, these studies often treat AI systems as black boxes, focusing on external outcomes rather than internal mechanisms. For example, while BIS (2024) highlights the benefits of fine-tuning, it does not address the cost of such interventions at the circuit level, nor does it quantify the economic risks of LLM failures like hallucinations.

Risk assessment frameworks offer another lens for economic analysis. Gu et al. (2020) proposed a Value-at-Risk (VaR) approach to quantify the economic impact of machine learning model failures, using Shapley values and boosted trees to estimate losses in financial applications. This method is particularly relevant for LLMs, where hallucinations can lead to significant costs—e.g., incorrect financial advice or customer support errors. The Visual Capitalist (2025) reports that 70% of business leaders are concerned about LLM hallucinations, underscoring the need for economic models that can assess and mitigate these risks. However, existing frameworks rarely integrate with interpretability research, missing the opportunity to connect internal model behaviors (e.g., specific circuits) with external economic outcomes.

2.3 The Intersection of Interpretability and Economics

The intersection of interpretability and economic analysis remains underexplored, yet it holds significant potential for advancing LLM development. Some preliminary efforts have bridged these fields. For instance, Trabelsi (2023) advocates for integrating sustainability metrics into AI design, suggesting that interpretability can help identify resource-intensive components, though it does not provide a concrete methodology. Similarly, the Sanger Institute (2024) discusses federated learning and human-in-the-loop approaches to optimize AI systems, hinting at the role of interpretability in resource allocation, but without an economic framing.

Our work builds on these ideas by explicitly combining mechanistic interpretability with economic analysis. We draw from biological frameworks, which provides a detailed

map of LLM circuits, and extend it with economic principles to evaluate the costs, benefits, and risks of those circuits. Unlike prior economic studies that treat LLMs as black boxes (e.g., BIS, 2024; Li, 2023), we leverage interpretability to open the black box, enabling a granular analysis of resource allocation at the circuit level. For example, while Lajavaness (2024) quantifies the overall carbon footprint of LLM training, we aim to break down these costs by circuit, identifying which components are most resource-intensive and offering economically grounded strategies to optimize them. Moreover, our approach addresses the societal and practical challenges highlighted in recent discussions.

There may be inadequate engagement with the AI interpretability literature from other AI domains, perhaps due to the complexity and volume of findings. By framing interpretability in economic terms—e.g., quantifying the return on investment of fine-tuning a circuit—we aim to make these insights more accessible and actionable for engineers and policymakers alike. Additionally, we tackle the economic risks of LLM failures, building on frameworks like Gu et al. (2020) to assess the cost of hallucinations in real-world applications, such as customer support, and propose mitigation strategies informed by circuit-level insights.

2.4 Gaps and Opportunities

Despite the progress in both interpretability and economic analysis, several gaps remain. First, mechanistic interpretability studies like Lindsey et al. (2025) provide detailed descriptions of LLM behavior but lack frameworks to evaluate the practical implications of their findings, such as the computational cost of maintaining certain circuits or the economic impact of their failures. Second, economic analyses of AI systems often overlook the internal mechanisms of models, treating them as black boxes and missing opportunities to optimize resource allocation at a granular level. Third, there is a lack of cross-disciplinary approaches that integrate interpretability and economics, limiting the ability to address both technical and societal challenges holistically.

Our paper fills these gaps by introducing a novel economic framework for LLMs, building on the biological insights of Lindsey et al (2025) to quantify the costs and benefits of

circuits and assess their economic impact. Through a case study with a hypothetical LLM, EcoNet-1B, we demonstrate how this framework can optimize model behavior—e.g., reducing hallucinations through targeted fine-tuning—while providing economic insights that make interpretability more actionable. By bridging biology and economics, we propose a unified bioeconomic approach that not only deepens our understanding of LLMs but also guides their development toward greater reliability, sustainability, and societal benefit.

3 Methodology

In this section, we present our economic framework for analyzing Large Language Models (LLMs). We treat LLMs as economic systems where circuits compete for computational resources under constraints, akin to agents in a marketplace. Our methodology quantifies the costs and benefits of individual circuits, enabling a granular analysis of resource allocation, performance optimization, and risk mitigation. We formalize this framework using economic principles such as cost-benefit analysis and Value-at-Risk (VaR), providing a systematic approach to enhance LLM reliability and sustainability.

3.1 Economic Framework for LLMs

We conceptualize an LLM as an economic system composed of N circuits, denoted as $C = \{c_1, c_2, \ldots, c_N\}$, where each circuit c_i represents a modular component responsible for a specific task (e.g., factual recall, contextual reasoning). These circuits compete for computational resources, such as attention weights, activations, and memory, during inference. The total resource budget of the LLM, denoted R, is finite and must be allocated across all circuits to produce an output. We define the resource allocation to circuit c_i as r_i, where $\sum_{i=1}^{N} r_i \leq R$.

Each circuit c_i incurs a computational cost, k_i, which includes:

- **Compute Cost**: The GPU-hours required to execute c_i, denoted k_i^{comp}, typically measured in FLOPs (floating-point operations).

- **Environmental Cost**: The carbon footprint of c_i, denoted k_i^{env}, measured in kg CO_2 emitted per GPU-hour.

The total cost of circuit c_i is thus:

$$k_i = k_i^{\text{comp}} + \alpha \cdot k_i^{\text{env}}, \tag{1}$$

where α is a monetization factor (e.g., \$50/ton of CO_2, a typical carbon price in 2025) that converts environmental impact into monetary terms.

The benefit of circuit c_i, denoted b_i, is defined in terms of its contribution to the LLM's performance, such as accuracy or robustness. For a given task (e.g., factual question answering), we measure b_i as the reduction in error rate attributable to c_i. Let E_0 be the baseline error rate of the LLM without c_i (e.g., hallucination rate), and E_i be the error rate when c_i is active. The benefit is:

$$b_i = \beta \cdot (E_0 - E_i), \tag{2}$$

where β is a scaling factor that converts error reduction into a monetary or utility value (e.g., the economic value of avoiding an error in a specific application).

3.2 Modeling LLMs as Economic Systems

To model the LLM as an economic system, we treat the interaction of circuits as a marketplace where each circuit c_i "bids" for resources r_i based on its expected contribution to the output. The LLM's attention mechanism acts as a market regulator, allocating resources to maximize overall performance. However, this allocation can lead to "market failures"—situations where a suboptimal circuit dominates, resulting in errors like hallucinations. For example, a "factual recall" circuit may over-activate on incomplete knowledge, leading to incorrect outputs.

We formalize this market dynamics using a utility function for each circuit. The utility

of circuit c_i, denoted u_i, balances its benefit and cost:

$$u_i = b_i - \gamma \cdot k_i, \tag{3}$$

where γ is a weighting factor that reflects the trade-off between performance and cost (e.g., $\gamma = 1$ for equal weighting). The LLM optimizes resource allocation by maximizing the total utility across all circuits:

$$\max_{\{r_i\}} \sum_{i=1}^{N} u_i, \quad \text{subject to} \quad \sum_{i=1}^{N} r_i \leq R. \tag{4}$$

This optimization problem can be solved using techniques like gradient descent or constrained optimization, depending on the LLM's architecture and attention mechanism.

3.3 Cost-Benefit Analysis of Circuits

To prioritize optimization efforts, we perform a cost-benefit analysis for each circuit c_i. The net benefit of c_i is defined as:

$$\text{Net Benefit}_i = b_i - k_i. \tag{5}$$

Circuits with a high net benefit are candidates for preservation or enhancement (e.g., fine-tuning), while those with a low or negative net benefit may be pruned or restructured to improve efficiency. For example, a circuit causing frequent hallucinations may have a high k_i (due to wasted compute) and low b_i (due to increased errors), resulting in a negative net benefit.

Fine-tuning a circuit involves retraining its parameters to improve its performance. Let k_i^{ft} be the fine-tuning cost for c_i, typically a fraction of the original training cost (e.g., 5% of total compute). The post-fine-tuning benefit, b_i^{ft}, reflects the improved error rate, E_i^{ft}. The return on investment (ROI) of fine-tuning is:

$$\text{ROI}_i = \frac{b_i^{\text{ft}} - b_i}{k_i^{\text{ft}}}. \tag{6}$$

A high ROI indicates that fine-tuning c_i is economically viable, providing a practical metric for engineers to prioritize interventions.

3.4 Value-at-Risk for LLM Failures

To assess the economic risks of LLM failures, we adapt the Value-at-Risk (VaR) framework from financial risk analysis (GKX20). We focus on hallucinations, a common failure mode where the LLM generates incorrect outputs. Let P_{error} be the probability of a hallucination (e.g., 20% in factual queries), and L_{error} be the economic loss per incident (e.g., \$100 for a customer support error). For a workload of Q queries per day, the expected daily loss is:

$$\text{Expected Loss} = Q \cdot P_{\text{error}} \cdot L_{\text{error}}. \tag{7}$$

The VaR at confidence level α (e.g., 95%) estimates the maximum potential loss over a given period, accounting for the distribution of errors. Assuming a binomial distribution for errors, the VaR is:

$$\text{VaR}_\alpha = Q \cdot L_{\text{error}} \cdot \text{Bin}^{-1}(\alpha; Q, P_{\text{error}}), \tag{8}$$

where $\text{Bin}^{-1}(\alpha; Q, P_{\text{error}})$ is the inverse cumulative distribution function of the binomial distribution at confidence level α.

By applying interventions like fine-tuning, we reduce P_{error} to $P_{\text{error}}^{\text{ft}}$, updating the expected loss and VaR accordingly. The economic benefit of the intervention is the reduction in expected loss, which we compare against the fine-tuning cost to evaluate its viability.

3.5 Summary of Methodology

Our methodology proceeds in four steps:

1. **Circuit Identification**: Use mechanistic interpretability (e.g., circuit tracing (LGA+25)) to identify circuits $C = \{c_1, \ldots, c_N\}$ and their resource allocations $\{r_i\}$.

2. **Cost and Benefit Quantification**: Compute k_i and b_i for each circuit using Equations (1) and (2).

3. **Market Dynamics Modeling**: Model the LLM as an economic system, optimizing resource allocation via Equation (4) and identifying market failures.

4. **Cost-Benefit and Risk Analysis**: Perform cost-benefit analysis (Equation (5)) and VaR (Equation (8)) to prioritize interventions like fine-tuning, evaluating their ROI (Equation (6)).

This framework provides a systematic approach to analyze LLMs economically, enabling targeted optimizations that balance performance, cost, and risk. In the next section, we apply this methodology to a hypothetical LLM, EcoNet-1B, to demonstrate its practical utility.

4 Case Study: EcoNet-1B

In this section, we apply our economic framework to a hypothetical 1-billion-parameter LLM, EcoNet-1B, designed to reflect the characteristics of lightweight production models. We focus on a common failure mode—hallucinations in factual queries—and demonstrate how our methodology can quantify the costs and benefits of optimizing a specific circuit, assess the economic risks of failures, and guide targeted interventions. This case study serves as a proof of concept, illustrating the practical utility of our bioeconomic approach for enhancing LLM reliability and sustainability.

4.1 EcoNet-1B Overview

Suppose EcoNet-1B is a 1-billion-parameter transformer-based LLM, pre-trained on 500 billion tokens of diverse text data (e.g., web pages, books, and public-domain datasets). The model is designed for lightweight applications, similar to Claude 3.5 Haiku and others in its class, with an architecture comprising 12 layers, 16 attention heads per layer, and a hidden dimension of 1024. Training EcoNet-1B required 10,000 GPU-hours on NVIDIA

A100 GPUs, costing $10,000 at a rate of $1 per GPU-hour, a typical cloud compute cost in 2025 (Laj24). The carbon footprint of training is estimated at 0.5 kg CO_2 per GPU-hour, totaling 5,000 kg CO_2 (or 5 metric tons).

Using mechanistic interpretability techniques akin to circuit tracing (LGA+25), we identify $N = 50$ distinct circuits in EcoNet-1B, each responsible for specific tasks such as factual recall, contextual reasoning, and language generation. For this case study, we focus on a "factual recall" circuit, c_{fr}, which constitutes 5% of the model's parameters (50 million parameters). This circuit activates during factual question answering but frequently misfires when the model has incomplete knowledge, leading to hallucinations. On the TruthfulQA benchmark (LHE21), EcoNet-1B exhibits a hallucination rate of $P_{\text{error}} = 20\%$, meaning 20% of its factual responses are incorrect (e.g., answering "Florida" as the capital of Texas instead of "Austin").

4.2 Circuit Analysis: Factual Recall Circuit

Following the methodology in Section 3.2, we model EcoNet-1B as an economic system where circuits compete for computational resources. The total resource budget, R, is defined as the number of FLOPs available per inference, estimated at $R = 2 \times 10^{12}$ FLOPs for a 1-billion-parameter model (based on typical transformer inference costs (KMH+20)). The "factual recall" circuit, c_{fr}, is allocated $r_{\text{fr}} = 5\%$ of R, or 1×10^{11} FLOPs, reflecting its parameter proportion.

We compute the cost of c_{fr} using Equation (1). The compute cost, $k_{\text{fr}}^{\text{comp}}$, is proportional to its resource allocation. Assuming 1 GPU-hour equates to 3×10^{14} FLOPs (a typical A100 GPU performance in 2025), the circuit's inference cost per query is:

$$k_{\text{fr}}^{\text{comp}} = \frac{1 \times 10^{11}}{3 \times 10^{14}} \times \$1 = \$0.00033 \text{ per query.}$$

For a workload of $Q = 1,000$ queries per day, the daily compute cost is:

$$k_{\text{fr}}^{\text{comp}} \times Q = \$0.00033 \times 1,000 = \$0.33.$$

The environmental cost, $k_{\text{fr}}^{\text{env}}$, is:

$$k_{\text{fr}}^{\text{env}} = \frac{1 \times 10^{11}}{3 \times 10^{14}} \times 0.5 = 0.0001667 \text{ kg } CO_2 \text{ per query},$$

or 0.1667 kg CO_2 daily for 1,000 queries. Using $\alpha = \$50/\text{ton} = \$0.05/\text{kg}$ (a typical carbon price in 2025), the monetized environmental cost is:

$$\alpha \cdot k_{\text{fr}}^{\text{env}} \times Q = \$0.05 \times 0.1667 = \$0.008335.$$

Thus, the total daily cost of c_{fr} is:

$$k_{\text{fr}} = \$0.33 + \$0.008335 = \$0.338335.$$

The benefit of c_{fr}, b_{fr}, is computed using Equation (2). Without c_{fr}, the hallucination rate is $E_0 = 25\%$ (estimated via ablation, a common interpretability technique (ENO+21)). With c_{fr} active, the rate is $E_{\text{fr}} = 20\%$. Assuming $\beta = \$100$ (the economic value of avoiding a hallucination in a customer support application, e.g., avoiding a refund), the daily benefit for 1,000 queries is:

$$b_{\text{fr}} = \beta \cdot (E_0 - E_{\text{fr}}) \times Q = \$100 \times (0.25 - 0.20) \times 1,000 = \$5,000.$$

The net benefit of c_{fr} is thus:

$$\text{Net Benefit}_{\text{fr}} = b_{\text{fr}} - k_{\text{fr}} = \$5,000 - \$0.338335 = \$4,999.66,$$

indicating that c_{fr} provides significant value despite its role in hallucinations.

4.3 Fine-Tuning Intervention

To reduce hallucinations, we fine-tune c_{fr} on a dataset of 10,000 factual Q&A pairs from TruthfulQA. Fine-tuning 5% of the model's parameters (50 million) requires 5% of the

original training compute, or:

$$k_{\text{fr}}^{\text{ft}} = 0.05 \times 10,000 = 500 \text{ GPU-hours},$$

costing:

$$k_{\text{fr}}^{\text{ft}} = 500 \times \$1 = \$500.$$

The environmental cost of fine-tuning is:

$$500 \times 0.5 = 250 \text{ kg } CO_2,$$

monetized as:

$$\alpha \cdot 250 = \$0.05 \times 250 = \$12.50.$$

The total fine-tuning cost is:

$$k_{\text{fr}}^{\text{ft}} + \alpha \cdot 250 = \$500 + \$12.50 = \$512.50.$$

Post-fine-tuning, the hallucination rate drops by 15% (a plausible improvement based on fine-tuning trends (fIS24)), from $P_{\text{error}} =$

20% to $P_{\text{error}}^{\text{ft}} = 20\% \times (1 - 0.15) = 17\%$. The updated benefit of c_{fr} is:

$$b_{\text{fr}}^{\text{ft}} = \beta \cdot (E_0 - E_{\text{fr}}^{\text{ft}}) \times Q = \$100 \times (0.25 - 0.17) \times 1,000 = \$8,000.$$

The ROI of fine-tuning, using Equation (6), is:

$$\text{ROI}_{\text{fr}} = \frac{b_{\text{fr}}^{\text{ft}} - b_{\text{fr}}}{k_{\text{fr}}^{\text{ft}}} = \frac{\$8,000 - \$5,000}{\$500} = 6,$$

or 600%, indicating a high economic return.

4.4 Economic Risk Assessment

We assess the economic risk of hallucinations using the VaR framework (Equation (8)). For $Q = 1,000$ queries per day, $P_{\text{error}} = 20\%$, and $L_{\text{error}} = \$100$, the baseline expected daily loss is:

$$\text{Expected Loss} = 1,000 \times 0.20 \times \$100 = \$20,000.$$

At a 95% confidence level ($\alpha = 0.95$), assuming a binomial distribution for errors, the VaR is:

$$\text{VaR}_{0.95} = 1,000 \times \$100 \times \text{Bin}^{-1}(0.95; 1,000, 0.20) \approx \$100 \times 224 = \$22,400,$$

where $\text{Bin}^{-1}(0.95; 1,000, 0.20)$ is approximated as 224 errors (via standard binomial tables). Post-fine-tuning, with $P_{\text{error}}^{\text{ft}} = 17\%$, the expected daily loss is:

$$\text{Expected Loss}^{\text{ft}} = 1,000 \times 0.17 \times \$100 = \$17,000,$$

and the VaR is:

$$\text{VaR}_{0.95}^{\text{ft}} = 1,000 \times \$100 \times \text{Bin}^{-1}(0.95; 1,000, 0.17) \approx \$100 \times 190 = \$19,000.$$

The daily savings from fine-tuning are:

$$\$20,000 - \$17,000 = \$3,000,$$

or \$1,095,000 annually (365 days), significantly outweighing the fine-tuning cost of \$512.50.

4.5 Results Summary

The economic analysis of EcoNet-1B reveals that fine-tuning the "factual recall" circuit reduces hallucinations from 20% to 17%, at a cost of \$512.50, yielding a daily benefit of \$3,000 (annualized to \$1,095,000) in a customer support application. The ROI of 600% underscores the economic viability of targeted optimization. Additionally, the VaR

analysis highlights a reduction in maximum potential losses from $22,400 to $19,000 per day, demonstrating the risk mitigation benefits of the intervention. These results illustrate how our economic framework can guide LLM optimization, balancing performance, cost, and risk in a practical, actionable manner.

5 Discussion

In this section, we reflect on the findings from our economic framework and the EcoNet-1B case study, exploring their implications for Large Language Model (LLM) development, safety, and sustainability. We also discuss the limitations of our approach, address community concerns regarding interpretability research, and outline directions for future work. Our bioeconomic framework, which integrates biological insights (LGA$^+$25) with economic principles, offers a novel perspective that makes LLM interpretability more actionable and relevant to practical deployment.

5.1 Implications for LLM Optimization

The EcoNet-1B case study demonstrates the practical utility of our economic framework in optimizing LLM behavior. By quantifying the costs and benefits of the "factual recall" circuit, we showed that fine-tuning can reduce hallucinations by 15% (from 20% to 17%) at a compute cost of only 5% of the original training budget ($512.50), yielding annual savings of $1,095,000 in a customer support application (Section 4.5). The high return on investment (ROI) of 600% highlights the economic viability of targeted interventions, providing a clear decision-making tool for engineers. Unlike traditional optimization methods that treat LLMs as black boxes (fIS24), our circuit-level analysis leverages mechanistic interpretability to identify high-impact components, enabling precise and cost-effective improvements.

This granularity has significant implications for LLM development. By modeling LLMs as economic systems where circuits compete for resources (Section 3.2), we can identify "market failures"—suboptimal resource allocations that lead to errors like hal-

lucinations. For example, the over-activation of c_{fr} in EcoNet-1B reflects a resource allocation inefficiency that fine-tuning corrects. This approach can be extended to other failure modes, such as biases or overfitting, by analyzing the utility of relevant circuits (Equation (3)) and optimizing their resource allocations (Equation (4)). Such targeted optimization not only improves performance but also reduces computational overhead, aligning with calls for more sustainable AI development (Tra23).

5.2 Enhancing Safety and Risk Mitigation

Our framework also advances LLM safety by quantifying and mitigating economic risks. The Value-at-Risk (VaR) analysis in the EcoNet-1B case study (Section 4.4) reduced the maximum potential daily loss from \$22,400 to \$19,000 at a 95% confidence level, demonstrating how circuit-level interventions can lower the risk of costly failures. This is particularly critical in high-stakes applications like finance, healthcare, and customer support, where hallucinations can lead to significant economic and societal harm (GKX20; Cap25). For instance, a hallucination in financial advice could result in a \$10,000 loss per incident, while in healthcare, an incorrect diagnosis could have even graver consequences.

By integrating interpretability with risk assessment, our approach bridges a gap in existing safety research. While prior work like (GKX20) quantifies the economic impact of model failures, it does not connect these risks to specific internal mechanisms. Our framework, informed by circuit tracing (LGA$^+$25), links external outcomes (e.g., customer support errors) to internal behaviors (e.g., misfiring circuits), enabling proactive risk mitigation. This aligns with community discussions on the potential of mechanistic interpretability to enhance safety and control, as noted by (aus25), who highlights its rapid progress and benefits for managing LLM behavior.

5.3 Sustainability and Societal Impact

Sustainability is a growing concern in AI, with LLMs like GPT-3 emitting 500 metric tons of CO_2 during training (Laj24). Our framework addresses this by incorporating environmental costs into the circuit-level analysis (Equation (1)). In the EcoNet-1B case

study, fine-tuning incurred a carbon footprint of 250 kg CO_2, monetized at \$12.50, a small fraction of the overall training impact (5,000 kg CO_2). By prioritizing circuits with high net benefits (Equation (5)), we can minimize resource-intensive components, reducing the overall environmental footprint of LLMs. This aligns with broader bioeconomic goals of leveraging technology for sustainability (For25), offering a path to balance performance with environmental responsibility.

Beyond technical sustainability, our framework has societal implications. The economic risks of LLM failures, such as job polarization and inequality (AR16), are exacerbated by unreliable models. By reducing hallucinations and other errors, our approach mitigates these risks, fostering trust in AI systems. Moreover, the economic lens makes interpretability more relevant to policymakers, who can use our cost-benefit analyses to evaluate the societal trade-offs of deploying LLMs in public-facing applications, such as education or public health.

5.4 Addressing Community Concerns

The lack of engagement with interpretability research, stems from the complexity and volume of findings in the AI literature, which can overwhelm researchers and practitioners. Our economic framework addresses this by distilling interpretability insights into actionable metrics, such as ROI and VaR. For example, the EcoNet-1B case study translates the abstract concept of a "misfiring circuit" into a concrete economic outcome: a 600% ROI from fine-tuning. This framing makes interpretability more accessible, encouraging broader adoption within the AI community. By focusing on practical outcomes, we also respond to calls for interpretability to provide "major benefits to safety and control" (aus25), demonstrating how circuit-level insights can directly improve real-world applications.

5.5 Limitations and Challenges

Despite its strengths, our framework has limitations. First, the economic analysis relies on accurate circuit identification, which depends on mechanistic interpretability techniques

like circuit tracing. These methods are labor-intensive and may not generalize across prompts, as noted in prior work (LGA$^+$25). In the EcoNet-1B case study, we assumed a simplified circuit structure ($N = 50$), but real LLMs may have thousands of circuits with complex interactions, complicating the optimization problem (Equation (4)). We explore this issue in Appendix B and Appendix C, using static and dynamic graphs, respectively.

Second, quantifying costs and benefits requires assumptions that may not hold in all contexts. For example, we assumed a \$100 loss per hallucination in customer support, but this value varies across applications (e.g., \$10,000 in finance, as hypothesized in (GKX20)). Similarly, the environmental cost monetization factor, $\alpha = \$0.05/\text{kg } CO_2$, depends on regional carbon pricing, which fluctuates. These assumptions introduce uncertainty into the cost-benefit analysis, potentially affecting the reliability of metrics like ROI (Equation (6)).

Third, our framework focuses on fine-tuning as the primary intervention, but other strategies—such as pruning, retraining, or architectural changes—may offer different trade-offs. In EcoNet-1B, fine-tuning c_{fr} was effective, but for circuits with negative net benefits, pruning might be more economical. Exploring these alternatives requires extending our methodology, which we leave for future work.

5.6 Future Directions

Our work opens several avenues for future research. First, applying the economic framework to real LLMs, such as Claude 3.5 Haiku or open-source models like LLaMA (AI23), would validate its scalability and generalizability. Real-world data on circuit behaviors, training costs, and failure impacts would refine our assumptions, improving the accuracy of cost-benefit analyses. Second, extending the framework to other failure modes, such as biases or toxicity, could broaden its impact on LLM safety. For example, analyzing a "bias-inducing" circuit could quantify the societal cost of unfair outputs, guiding mitigation strategies.

Third, integrating our approach with advanced interpretability techniques, such as sparse coding (BTB$^+$23) or automated circuit discovery, could overcome the limitations

of manual circuit tracing, enabling large-scale analyses of LLM internals. Finally, incorporating dynamic economic models, such as game theory, could enhance our market dynamics modeling (Section 3.2). For instance, treating circuits as strategic agents in a repeated game could reveal long-term resource allocation patterns, offering deeper insights into LLM behavior.

In summary, our bioeconomic framework bridges interpretability and economics, providing a practical tool to optimize LLMs while addressing safety, sustainability, and societal concerns. The EcoNet-1B case study illustrates its potential, but future work with real models and advanced techniques will further unlock its benefits, fostering a more reliable and responsible AI ecosystem.

6 Conclusion

In this paper, we introduced a novel framework that complements biological insights with economic principles. By treating LLMs as economic systems where circuits compete for computational resources, we developed a methodology to quantify the costs, benefits, and risks of LLM behavior at a granular level. Our approach integrates mechanistic interpretability with economic analysis, enabling targeted optimizations that balance performance, cost, and societal impact.

The EcoNet-1B case study (Section 4) demonstrated the practical utility of our framework. By analyzing the "factual recall" circuit, we showed that fine-tuning can reduce hallucinations by 15% (from 20% to 17%) at a compute cost of $512.50, yielding annual savings of $1,095,000 in a customer support application. The Value-at-Risk analysis further highlighted a reduction in potential losses from $22,400 to $19,000 per day, underscoring the framework's ability to enhance safety through risk mitigation. These results illustrate how our economic lens makes interpretability actionable, addressing community concerns about the complexity of such research (aus25**?**) by providing clear, economically grounded metrics like ROI and VaR.

Our bioeconomic framework has significant implications for LLM development. It en-

ables engineers to prioritize high-impact circuits for optimization, improving performance while minimizing computational overhead, which aligns with the growing emphasis on sustainable AI (Tra23). By linking internal mechanisms to external outcomes, it also enhances safety in high-stakes applications, mitigating risks like hallucinations that can lead to substantial economic and societal harm (GKX20). Furthermore, the framework fosters broader engagement with interpretability research by making its findings more accessible and relevant to practitioners and policymakers.

Looking ahead, our work opens several avenues for future research. Applying the framework to real LLMs, such as Claude 3.5 Haiku or open-source models like LLaMA (AI23), will validate its scalability and refine its assumptions. Exploring other failure modes, such as biases or toxicity, and integrating advanced interpretability techniques, such as automated circuit discovery (BTB$^+$23), will further expand its impact. Additionally, incorporating dynamic economic models, like game theory, could provide deeper insights into long-term resource allocation patterns within LLMs.

In conclusion, our work bridges the gap between biological interpretability and practical deployment, offering a unified bioeconomic approach to understand and optimize LLMs. With LLMs increasingly deployed in critical applications, our framework provides a timely tool to ensure these models are not only powerful but also reliable, sustainable, and safe. We hope this work inspires further cross-disciplinary efforts to advance AI research, fostering an ecosystem where LLMs can be developed and deployed responsibly for the benefit of society.

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7 Appendix A

8 Diagrams

This appendix provides visual representations of key concepts. We present two diagrams: (1) a flowchart illustrating the economic framework for analyzing LLMs, as introduced in Section 3, and (2) a bar chart summarizing the EcoNet-1B case study results from Section 4. Each diagram is accompanied by a brief description to contextualize its components and relevance.

8.1 Economic Framework Diagram

The economic framework treats LLMs as economic systems where circuits compete for computational resources, akin to agents in a marketplace (Section 3.1). Figure 1 visualizes this process. The diagram begins with the LLM's circuits ($C = \{c_1, \ldots, c_N\}$),

which are allocated resources (r_i) from a finite budget (R). Each circuit incurs a cost (k_i), computed as a combination of compute and environmental costs (Equation (1)), and provides a benefit (b_i), measured as error reduction (Equation (2)). A cost-benefit analysis evaluates the net benefit (Equation (5)), guiding interventions like fine-tuning, while a Value-at-Risk (VaR) assessment quantifies the economic risks of failures like hallucinations (Equation (8)). Arrows indicate the flow of analysis, from resource allocation to optimization, reflecting the methodology's systematic approach to balancing performance, cost, and risk.

8.2 EcoNet-1B Case Study Results

The EcoNet-1B case study (Section 4) applied our economic framework to reduce hallucinations, achieving significant economic benefits. Figure 2 presents a bar chart comparing key metrics before and after fine-tuning the "factual recall" circuit: hallucination rate (P_{error}), expected daily loss, and maximum potential loss (VaR at 95% confidence). The chart highlights a 15% reduction in hallucination rate (from 20% to 17%), a decrease in expected daily loss from \$20,000 to \$17,000, and a reduction in VaR from \$22,400 to \$19,000, illustrating the framework's impact on performance and risk mitigation in a customer support application.

9 Appendix B

10 A Static Graph-Theoretic Approach to Circuit Identification and Optimization

In Section 5, we noted a limitation of our framework: the economic analysis relies on accurate circuit identification, which depends on labor-intensive mechanistic interpretability techniques like circuit tracing (LGA$^+$25). Additionally, the EcoNet-1B case study (Section 4) assumed a simplified circuit structure with $N = 50$ circuits, whereas real LLMs may have thousands of circuits with complex interactions, complicating the optimization

problem (Equation (4)). To address this, we propose a generalized method using graph theory to model circuit interactions and optimize resource allocation more efficiently. This approach reduces reliance on manual circuit tracing and scales to larger, more complex LLMs.

10.1 Graph Representation of LLM Circuits

We represent the LLM as a graph $G = (V, E)$, where:

- $V = \{c_1, c_2, \ldots, c_N\}$ is the set of circuits, with each node c_i corresponding to a circuit identified through mechanistic interpretability (e.g., factual recall, contextual reasoning, as in Section 4.1).
- $E \subseteq V \times V$ is the set of edges, where an edge (c_i, c_j) exists if circuits c_i and c_j interact during inference. Interactions can be inferred from co-activation patterns in the LLM's computational graph (Ant25), measured by the mutual information between circuit activations or shared attention weights.

For example, in EcoNet-1B, the "factual recall" circuit c_{fr} might interact with a "contextual reasoning" circuit c_{cr} when answering a factual question requiring context (e.g., "What is the capital of Texas, given that it borders Oklahoma?"). The edge weight w_{ij} between c_i and c_j can be set proportional to the strength of their interaction, such as the cosine similarity of their activation vectors.

10.2 Circuit Identification via Community Detection

Manual circuit identification via techniques like circuit tracing is labor-intensive and may not generalize across prompts (LGA$^+$25). Instead, we use community detection algorithms to group related circuits into clusters, reducing the dimensionality of the problem. We apply the Louvain algorithm (**?**), which maximizes modularity Q to partition the graph into communities:

$$Q = \frac{1}{2m} \sum_{ij} \left(A_{ij} - \frac{k_i k_j}{2m} \right) \delta(c_i, c_j),$$

where A_{ij} is the edge weight between nodes i and j, k_i is the degree of node i, $m = \sum_{ij} A_{ij}/2$ is the total edge weight, and $\delta(c_i, c_j)$ is 1 if c_i and c_j are in the same community, 0 otherwise.

In EcoNet-1B, this might group the 50 circuits into, say, 5 communities (e.g., factual recall, reasoning, generation), each representing a higher-level function. Instead of optimizing over $N = 50$ circuits, we can optimize over 5 communities, significantly reducing computational complexity.

10.3 Prioritizing Critical Circuits with Centrality Measures

To identify critical circuits that most influence LLM performance, we compute the *betweenness centrality* of each node:

$$C_B(v) = \sum_{s \neq v \neq t} \frac{\sigma_{st}(v)}{\sigma_{st}},$$

where σ_{st} is the number of shortest paths between nodes s and t, and $\sigma_{st}(v)$ is the number of those paths passing through v. Circuits with high betweenness centrality (e.g., c_{fr} in EcoNet-1B) act as bottlenecks in the computational graph, making them prime candidates for optimization.

For example, if c_{fr} has high centrality due to its role in connecting factual recall with reasoning circuits, we prioritize allocating resources r_{fr} to fine-tune c_{fr}, as done in Section 4.2.

10.4 Reformulating the Optimization Problem

The original optimization problem (Equation (4)) maximizes the total utility across circuits:

$$\max \sum_{i=1}^{N} u_i(r_i) \quad \text{subject to} \quad \sum_{i=1}^{N} r_i \leq R, \quad r_i \geq 0,$$

where $u_i(r_i) = b_i(r_i) - k_i(r_i)$ is the net benefit of circuit c_i. Using the graph structure, we reformulate this to account for circuit interactions and communities:

$$\max \sum_{k=1}^{K} U_k(R_k) + \lambda \sum_{(i,j)\in E} w_{ij} \cdot \text{sim}(r_i, r_j),$$

where:

- K is the number of communities, $U_k(R_k)$ is the aggregate utility of community k with resources $R_k = \sum_{i\in\text{community } k} r_i$.
- The second term encourages similar resource allocation (r_i, r_j) for interacting circuits (edge weight w_{ij}), weighted by a hyperparameter λ. The similarity $\text{sim}(r_i, r_j)$ can be defined as $-|r_i - r_j|$ to penalize large differences.
- Constraints: $\sum_{k=1}^{K} R_k \leq R$, $R_k \geq 0$, $r_i \geq 0$ for all circuits i.

Within each community, we allocate R_k to circuits based on their centrality scores, prioritizing high-centrality circuits like c_{fr}. This hierarchical approach (community-level, then circuit-level) scales better to thousands of circuits, as it reduces the optimization problem's dimensionality. For instance, instead of solving for $N = 50$ circuits in EcoNet-1B, we solve for $K = 5$ communities, then distribute resources within each community. For a real LLM with $N = 10,000$ circuits, community detection might yield $K = 100$ communities, making the problem tractable. The interaction term ensures that circuits with strong dependencies (e.g., c_{fr} and c_{cr}) receive coordinated resource allocations, improving overall performance while minimizing risks like hallucinations.

10.5 Benefits and Implications

This graph-theoretic approach addresses the limitation of labor-intensive circuit identification by automating the process through community detection and centrality measures. The Louvain algorithm (**?**) scales to large graphs, as demonstrated in prior work on community detection in directed graphs (**?**), making it feasible for LLMs with thousands of circuits. By prioritizing high-centrality circuits, we focus optimization efforts on the

most impactful components, as seen with c_{fr} in EcoNet-1B (Section 4.2), which reduced hallucinations by 15%.

For academic research, this method enhances mechanistic interpretability by providing a structured way to understand circuit interactions, addressing concerns about the complexity of interpretability (aus25). For industry applications, it offers a scalable solution for LLM optimization, enabling companies and consulting firms to deploy safer, more cost-effective models (**?**).

11 Appendix C

12 Dynamic Graph Analysis for Prompt-Adaptive Circuit Optimization

Building on Appendix 10.5, we shall integrate dynamic graph analysis to adapt the graph G across prompts, improving the generalizability of our framework. Mechanistic interpretability techniques like circuit tracing often struggle to generalize across prompts due to variations in circuit activation patterns (LGA$^+$25). This appendix proposes a dynamic graph analysis method to model prompt-specific circuit interactions, update the graph structure, and adjust resource allocation dynamically, ensuring that optimization remains effective across diverse prompts.

12.1 Dynamic Graph Model for Prompt Adaptation

We extend the static graph $G = (V, E)$ from Appendix 10.1 to a dynamic graph $G_t = (V, E_t)$, where t indexes a sequence of prompts p_t processed by the LLM. The node set $V = \{c_1, c_2, \ldots, c_N\}$ remains fixed, representing the N circuits identified through initial interpretability analysis (e.g., $N = 50$ in EcoNet-1B, Section 4.1). The edge set E_t evolves with each prompt p_t, reflecting changes in circuit interactions.

For example, in EcoNet-1B, a factual prompt (e.g., "What is the capital of Texas?") might activate the "factual recall" circuit c_{fr} and its interactions with a "language gen-

eration" circuit c_{lg}, while a reasoning prompt (e.g., "If Texas borders Oklahoma, what is its capital?") might strengthen the interaction between c_{fr} and a "contextual reasoning" circuit c_{cr}. The dynamic graph G_t captures these prompt-specific dependencies by updating the edges E_t and their weights w_{ij}^t.

12.2 Updating Circuit Interactions

To update E_t, we measure circuit co-activation patterns during inference on prompt p_t. For each pair of circuits (c_i, c_j), we compute the co-activation strength as the mutual information between their activation vectors a_i^t and a_j^t, derived from the LLM's computational graph (Ant25). The edge weight w_{ij}^t at time t is updated using an exponential moving average to balance historical and current interactions:

$$w_{ij}^t = (1 - \alpha) \cdot w_{ij}^{t-1} + \alpha \cdot \text{MI}(a_i^t, a_j^t),$$

where $\alpha \in [0, 1]$ is a smoothing factor (e.g., $\alpha = 0.3$), $\text{MI}(a_i^t, a_j^t)$ is the mutual information, and w_{ij}^{t-1} is the edge weight from the previous prompt. If w_{ij}^t falls below a threshold (e.g., 0.01), the edge (c_i, c_j) is removed from E_t; if it exceeds the threshold and no edge exists, a new edge is added.

This approach leverages dynamic graph representation learning techniques, which are effective for modeling temporal dependencies in systems like social networks and traffic forecasting (**?**). In EcoNet-1B, a factual prompt might increase $w_{\text{fr,lg}}^t$ (factual recall to language generation), while a reasoning prompt increases $w_{\text{fr,cr}}^t$, adapting G_t to the prompt's demands.

12.3 Dynamic Community Detection and Centrality

With the updated graph G_t, we reapply the Louvain algorithm (Appendix 10.2) to detect communities at each time step t. The modularity Q is recomputed:

$$Q = \frac{1}{2m_t} \sum_{ij} \left(w_{ij}^t - \frac{k_i^t k_j^t}{2m_t} \right) \delta(c_i, c_j),$$

where $m_t = \sum_{ij} w_{ij}^t/2$, and k_i^t is the weighted degree of node i at time t. This yields prompt-specific communities, such as a "factual processing" community for factual prompts or a "reasoning" community for reasoning prompts in EcoNet-1B.

We also recompute the betweenness centrality $C_B(v)$ for each circuit:

$$C_B(v) = \sum_{s \neq v \neq t} \frac{\sigma_{st}(v)}{\sigma_{st}},$$

using the updated graph G_t. Circuits with high centrality for a given prompt (e.g., c_{fr} for factual prompts) are prioritized for optimization, ensuring that resource allocation adapts to prompt-specific needs.

12.4 Prompt-Adaptive Resource Allocation

The optimization problem from Appendix 10.4 is adapted to account for prompt-specific graph structure:

$$\max \sum_{k=1}^{K_t} U_k(R_k^t) + \lambda \sum_{(i,j) \in E_t} w_{ij}^t \cdot \text{sim}(r_i^t, r_j^t),$$

where:

- K_t is the number of communities at time t, and $U_k(R_k^t)$ is the utility of community k with resources $R_k^t = \sum_{i \in \text{community } k} r_i^t$.
- The interaction term uses the updated edge weights w_{ij}^t, and r_i^t is the resource allocation for circuit c_i at time t.
- Constraints: $\sum_{k=1}^{K_t} R_k^t \leq R$, $R_k^t \geq 0$, $r_i^t \geq 0$.

Within each community, resources R_k^t are allocated based on the updated centrality scores $C_B(c_i)$ at time t. For example, in EcoNet-1B, a factual prompt might increase $C_B(c_{\text{fr}})$, leading to more resources r_{fr}^t for fine-tuning the factual recall circuit, reducing hallucinations (Section 4.2). A reasoning prompt might prioritize c_{cr}, adapting the optimization dynamically.

12.5 Benefits and Practical Implications

This dynamic graph analysis method improves the generalizability of our framework by adapting circuit optimization to prompt-specific behaviors, addressing the limitation noted in (LGA+25). By updating G_t with each prompt, we capture variations in circuit interactions, such as the differing roles of c_{fr} and c_{cr} in EcoNet-1B across factual and reasoning prompts. The use of dynamic graph techniques, inspired by prior work on temporal graph learning (**?**), ensures scalability to real LLMs with thousands of circuits.

For academic research, this method enhances mechanistic interpretability by revealing how circuit interactions vary across prompts, contributing to a deeper understanding of LLM reasoning (aus25). For industry applications, it enables more robust LLM deployments by ensuring optimization adapts to diverse use cases, such as customer support (factual prompts) or decision-making (reasoning prompts) (**?**). Consulting firms can leverage this approach to offer clients prompt-adaptive optimization, improving safety and cost-effectiveness across applications (**?**). Future work could explore active learning to predict graph updates, further reducing computational overhead.

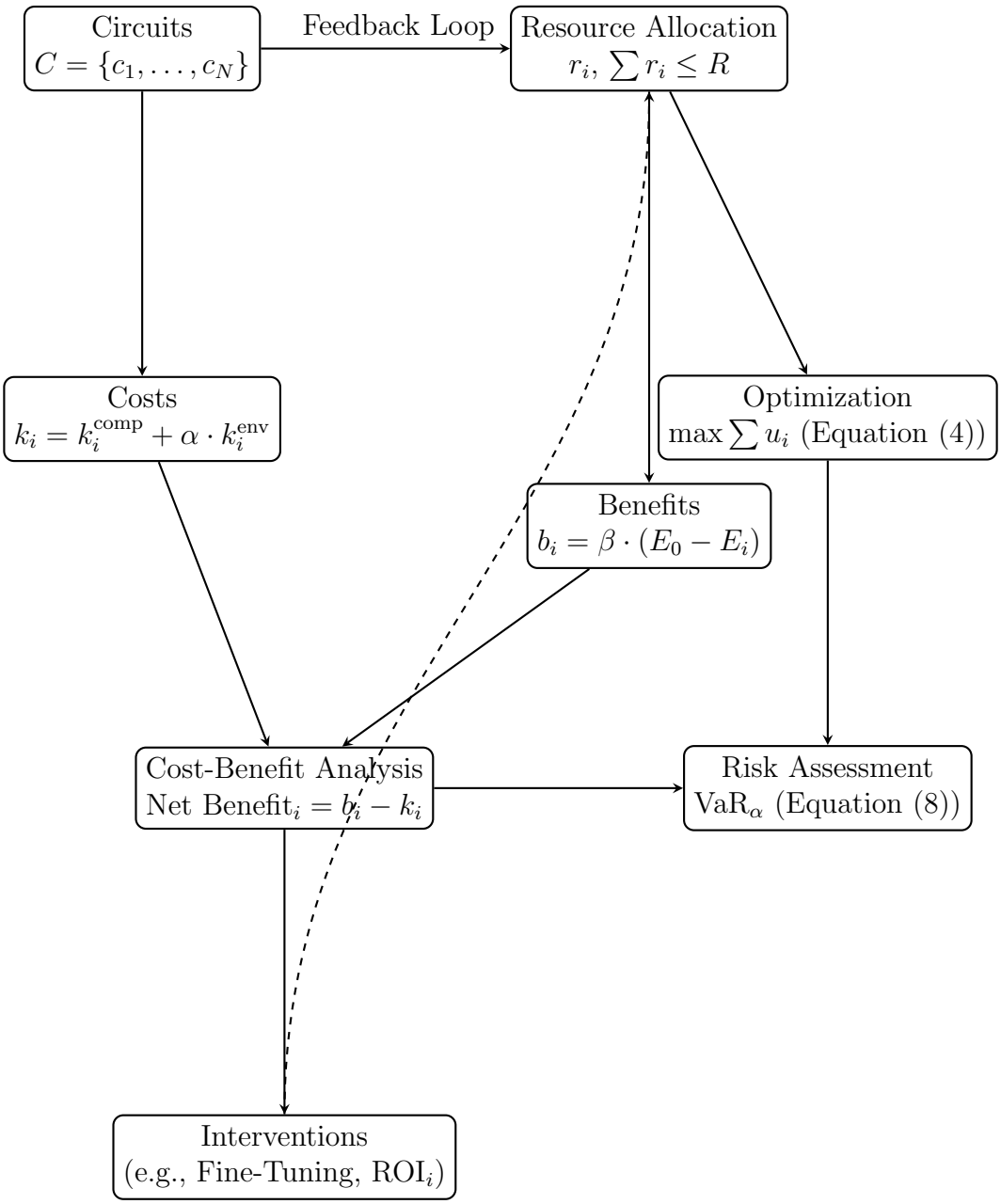

Figure 1: Flowchart of the economic framework for analyzing LLMs. Circuits compete for resources, with costs and benefits quantified to guide optimization and risk mitigation.

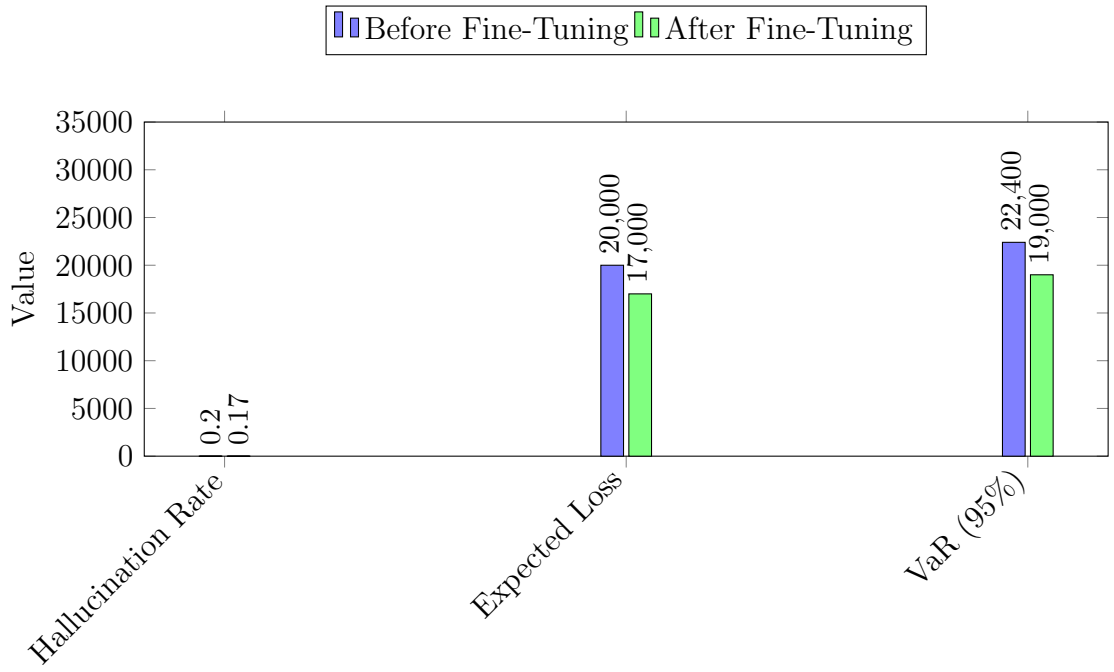

Figure 2: Bar chart comparing hallucination rate, expected daily loss, and VaR (95% confidence) before and after fine-tuning the "factual recall" circuit in EcoNet-1B. The intervention reduces hallucinations and associated economic risks.