The Economics of Organizational Coordination: A General Theory of the Firm

Kweku Opoku-Agyemang*

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Abstract

The theory of the firm explains organizational boundaries primarily through the relative costs of market exchange and hierarchical coordination. This paper develops a general theory in which productive activity can instead be coordinated through markets, hierarchies, or distributed networks. The framework introduces portable network capital (informational, relational, reputational, and distributional capabilities that can reside with agents independently of firms) and allows coordination technologies to exhibit increasing returns to network scale. We characterize the endogenous formation of networks and organizational boundaries and derive conditions under which technological progress changes firm scale, concentration, and the allocation of activity across ownership boundaries. A central result is that productive networks and firms need not coincide: declining costs of communication, reputation, and coordination can allow productive relationships to remain distributed while ownership becomes less vertically integrated. The theory therefore replaces the market-versus-hierarchy dichotomy with an endogenous continuum of organizational forms, from autonomous agents and temporary coalitions to networked organizations and integrated firms. Social media and artificial intelligence are consequential in this framework not as separate sectors, but as technologies that alter the costs, portability, and scalability of economic coordination.

*Machine Learning X Doing Incorporated. Email: kweku@machinelearningxdoing.com.

1 Introduction

Why do firms exist?

Since Coase (1937), this question has been central to the economics of organization. Economic activity can, in principle, be coordinated through decentralized market exchange: agents can contract with one another, respond to prices, and purchase inputs whenever they are needed. Yet much production is instead conducted within organizations in which authority, rather than prices, coordinates the allocation of resources. The existence, size, and boundaries of firms therefore represent a fundamental economic problem: under what conditions is it more efficient to coordinate productive activity through an organization than through market exchange?

The canonical theory of the firm provides several answers to this question. In the transaction-cost tradition, firms substitute authority for costly market transactions when the costs of using the price mechanism become sufficiently large (**?**Williamson, 1975). In theories of team production, organizations arise because coordinated production creates monitoring and free-riding problems that cannot be efficiently resolved through separate contracts (Alchian and Demsetz, 1972). In the property-rights approach, organizational boundaries determine the allocation of residual control rights over productive assets and thereby affect investment incentives under incomplete contracting (Grossman and Hart, 1986; Hart and Moore, 1990). Although these theories differ substantially in mechanism, they share a common structure. Organizational form is understood as a solution to a coordination problem in which alternative arrangements differ in their costs of exchange, monitoring, contracting, control, and ownership.

This paper argues that the canonical problem is more general than the market-versus-firm dichotomy through which it has traditionally been formulated. The fundamental economic question is not whether production should occur in markets or firms, but how heterogeneous productive capabilities should be coordinated. Markets and hierarchies are two important coordination architectures, but they need not exhaust the feasible set. A third architecture becomes increasingly consequential when information, reputation, productive capabilities, and coordination capacity can move across organizational boundaries: *distributed network coordination*. Under network coordination, agents can remain legally and economically distinct while participating in persistent or temporary structures for sharing information, establishing trust, combining complementary capabilities, and reaching markets. Such arrangements include neither conventional anonymous market exchange nor conventional hierarchical integration.

The distinction is consequential because the technological environment underlying the canonical theories has changed. Digital communication and social platforms have dramatically reduced the costs of discovering counterparties, transmitting information, establishing public reputations, and maintaining relationships across geographic and organizational boundaries. Market access can increasingly be obtained through an individual's own network rather than through affiliation with a large organization. Productive capabilities that were historically bundled inside firms can consequently become economically accessible to independent agents. At the same time, advances in artificial intelligence are reducing the cost of coordinating heterogeneous tasks, transforming information, generating intermediate outputs, and managing complex workflows. These developments do not merely increase the productivity of existing firms. They change the relative costs of alternative organizational architectures.

The theoretical implication is not that technology necessarily causes firms to become smaller, nor that markets necessarily replace hierarchies. Both conclusions are too strong. A reduction in coordination costs can make previously internal activities more efficiently distributed across independent agents, while simultaneously increasing the returns to organizational scale when coordination technologies complement assets that remain concentrated. The relevant question is therefore an equilibrium one: how does a change in the costs of market, network, and hierarchical coordination alter the optimal allocation of productive activity?

We develop a general framework to answer this question. The starting point is a reconstruction of the canonical theory of the firm as a coordination problem. Consider a collection of productive capabilities that must be combined to generate output. The planner, firm, or decentralized economy faces alternative mechanisms for coordinating these capabilities. In the canonical environment, the principal comparison is between decentralized exchange and internal organization. We generalize this problem by allowing productive capabilities to be coordinated through three distinct architectures: markets, networks, and hierarchies. Each architecture is characterized by a distinct set of coordination costs and productive advantages. Organizational boundaries then emerge endogenously from the allocation that minimizes the total cost of coordinating a given production process.

This formulation permits us to identify precisely which assumptions are required for the conventional market-versus-firm distinction to arise as an equilibrium outcome. In particular, the canonical framework effectively treats access to information, reputation, coordination, and productive resources as sufficiently localized that organizational affiliation materially affects an agent's ability to participate in production. When these resources become portable across organizational boundaries, that assumption no longer holds. We therefore introduce two concepts that are central to the extended framework.

The first is *portable network capital*. An agent's reputation, relationships, accumulated information, audience, distribution channels, and demonstrated reliability can constitute productive capital that is not necessarily owned by or embedded within a firm. Unlike many conventional organizational assets, such capital can accompany an agent across contractual and organizational boundaries. It can reduce search, trust, matching, and market-access costs independently of formal organizational membership. The portability of this capital changes the economic relationship between individual productive capacity and firm-level productive capacity.

The second is *general-purpose coordination capital*. Technologies that can perform or substantially reduce the costs of planning, information processing, communication, monitoring, translation, and task coordination effectively expand the amount of heterogeneous productive activity that a given coordinating agent can manage. Artificial intelligence is an especially important instance because its capabilities can be deployed across tasks rather than being tied to a single production process. In the model, however, artificial intelligence is not assumed to be uniquely transformative. It enters as a technology that changes the parameters governing coordination. The framework therefore applies to any technological development that systematically reduces the cost of coordinating distributed productive capabilities.

The central theoretical result is that the firm's boundary is not fundamentally a binary choice between market exchange and hierarchy. It is the endogenous boundary of a broader coordination problem. Depending on the relative costs and complementarities of the three architectures, equilibrium production can take the form of vertically integrated firms, decentralized market exchange, networked organizations, temporary coalitions, or hybrid structures combining internal and external coordination. Conventional firms emerge as a special case when network coordination is sufficiently costly or when the productive assets required for coordination are sufficiently organization-specific. Conversely, networked forms become optimal when portable network capital and coordination technologies sufficiently reduce the costs of coordinating independent agents.

This framework generates several implications that distinguish it from a simple technological extension of the existing literature. First, reductions in coordination costs can change the optimal *boundary* of the firm even when the underlying production technology is unchanged. Second, the relevant scale of an organization can diverge from the scale of the productive network in which it operates: a small organizational core may coordinate a substantially larger distributed production system. Third, technological progress can have opposing effects on organizational concentration. When coordination technologies substitute for internal hierarchy, they can reduce efficient firm size; when they complement concentrated assets or increase the returns to controlling a network, they can increase concentration. Fourth, individual agents' accumulated network capital can become a source of organizational power, creating a form of productive capital that is neither conventionally owned by firms nor reducible to human capital.

These results suggest a reinterpretation of the firm itself. The firm need not be understood primarily as a physical locus of production, a collection of employees, or a bundle of owned assets. These characteristics describe particular organizational forms rather than the general economic function of organization. At a more fundamental level, a firm is an *institutional architecture for coordinating productive capacity*. Its boundaries arise when internal coordination provides sufficient advantages relative to alternative architectures to justify bringing productive activity under a common organizational structure. The same theory can therefore encompass the vertically integrated industrial corporation, the platform-mediated enterprise, the networked organization, and emerging forms of highly automated production without treating any one of them as the defining form of the firm.

The contribution is thus not to propose that social media or artificial intelligence has made the traditional theory of the firm obsolete. Rather, the paper identifies the canonical market-versus-hierarchy framework as a special case of a more general theory of organizational coordination. The technologies associated with the twenty-first century matter because they alter the economic primitives from which organizational boundaries emerge: the portability of information and reputation, the accessibility of productive capabilities, the cost of matching and trust, and the capacity to coordinate heterogeneous activity. Once these primitives are allowed to vary, the familiar firm becomes one equilibrium form within a broader space of organizational architectures.

The remainder of the paper proceeds as follows. Section 2 reconstructs the canonical theory of the firm in a common mathematical framework and shows how the principal contributions of the transaction-cost, team-production, and property-rights traditions can be interpreted as special cases of a coordination problem. Section 3 identifies the assumptions under which the canonical market-versus-hierarchy result obtains and examines how developments in information, communication, reputation, and coordination technologies relax them. Section 4 introduces distributed network coordination and develops the concepts of portable network capital and general-purpose coordination capital. Section 5 presents the general equilibrium framework and derives the endogenous boundaries of markets, networks, and firms. Section 6 establishes the principal comparative statics and identifies conditions under which technological progress expands or contracts organizational scale and concentration. Section 7 relates the framework to the existing theories of the firm, organizational economics, networks, platforms, and technological change. Section 8 considers implications for the organization of production in an increasingly networked and AI-enabled economy. Section 9 concludes.

2 The Canonical Theory of the Firm as a Coordination Problem

The modern theory of the firm contains several distinct theoretical traditions, but these traditions can be represented within a common underlying problem. A set of productive activities must be coordinated to generate output. Coordination can occur through decentralized exchange, through an organizational structure in which decisions are allocated by authority, or through intermediate institutional arrangements. The economic problem is therefore to determine which allocation of productive activity minimizes the costs of coordination while preserving the productive gains from specialization and cooperation.

This section reconstructs the canonical theory of the firm in this common form. The purpose is not to collapse the differences between transaction-cost, team-production, and property-rights theories. Rather, it is to identify the common object over which those theories differ: the cost of coordinating productive activity under alternative organizational arrangements. This formulation will subsequently allow us to introduce coordination architectures that are not captured by the conventional market-versus-hierarchy dichotomy.

2.1 A General Coordination Problem

Consider an economy with a finite set of productive activities

$$\mathcal{A} = \{1, \ldots, n\}.$$

Activity i requires a productive capability k_i and contributes to output through a production function

$$Y = F(k_1, \ldots, k_n).$$

The productive capabilities may be embodied in individuals, physical assets, knowledge, or other productive resources. For the moment, their ownership and location are left unspecified. The central problem is how these capabilities are coordinated.

Let an organizational allocation be represented by a partition

$$\Pi = \{G_1, \ldots, G_m\}$$

of the set of activities $\mathcal{A}$. Each element G_j represents a collection of activities coordinated within a common organizational arrangement. The extreme case

$$\Pi^M = \{\{1\}, \ldots, \{n\}\}$$

corresponds to fully decentralized production, while

$$\Pi^H = \{\mathcal{A}\}$$

corresponds to complete hierarchical integration.

For a given allocation Π, let

$$C(\Pi)$$

denote the total cost of coordinating the associated productive activities. Let $R(\Pi)$ denote the productive return generated by the allocation. The organizational problem can then be written as

$$\Pi^* = \arg\max_{\Pi} \{R(\Pi) - C(\Pi)\}.$$

This formulation is deliberately general. It does not assume that coordination costs arise from any particular mechanism. They may result from bargaining, search, contracting, monitoring, information asymmetries, incomplete contracts, opportunism, communication failures, or the costs of exercising authority. What distinguishes theories of the firm is, in part, how these costs are specified.

The canonical theory can therefore be understood as imposing particular restrictions on the feasible organizational allocations and on the structure of $C(\Pi)$.

3 The Assumptions of the Canonical Framework and Their Relaxation

The canonical theory of the firm can be represented as a special case of the coordination problem developed in Section 2. Its central result follows when the relevant alternatives are restricted to decentralized market exchange and hierarchical organization, and when several properties of information, reputation, productive capabilities, and coordination capacity are treated as given. The purpose of this section is to make those assumptions explicit.

Doing so is important for two reasons. First, the emergence of new organizational forms cannot be established merely by observing that firms increasingly use digital technologies or artificial intelligence. A new theory requires a demonstration that the underlying economic conditions determining organizational boundaries have changed. Second, technological change does not render the canonical theories incorrect. Rather, it can change the parameters and feasible set of the coordination problem to which those theories apply.

We therefore distinguish between *technological changes in coordination costs* and *changes in the set of feasible coordination architectures*. The first modifies the relative efficiency of existing organizational forms. The second can make previously dominated forms economically viable. The distinction is central to the theory developed below.

3.1 The Canonical Assumptions

Consider the canonical organizational choice

$$s^* = \arg \min_{s \in \{M,H\}} C(s),$$

where M denotes market coordination and H denotes hierarchical coordination.

The binary structure of this problem is supported by several implicit assumptions.

3.1.1 Localized Information

The first assumption is that economically relevant information is costly to acquire, transmit, and coordinate across organizational boundaries. Information may be imperfectly observable, privately held, or costly to communicate. As a result, organizational proximity can provide an economically valuable mechanism for aggregating information.

Formally, let I_i denote information held by agent i, and let

$$c_I(d_{ij})$$

denote the cost of transmitting or coordinating information between agents i and j, where d_{ij} represents the effective organizational or relational distance between them. The canonical environment permits

$$\frac{\partial c_I}{\partial d_{ij}} > 0.$$

If information becomes sufficiently costly to transmit across organizational boundaries, bringing agents into a common organization can reduce coordination costs.

3.1.2 Localized Reputation

A second assumption concerns trust and reputation. In conventional markets, agents must determine whether potential counterparties are reliable, competent, and likely to honor agreements. Repeated interaction, organizational affiliation, legal enforcement, and intermediary institutions can reduce these costs.

Let r_i denote the reputation of agent i, and let ρ_{ij} denote the extent to which agent j can observe and rely upon i's reputation. In the canonical environment, reputation is often effectively local:

$$\rho_{ij} = g(\text{prior interaction}_{ij}, \text{institutional affiliation}_i, \ldots).$$

Consequently, an individual's reputation may provide limited value outside the set of relationships in which it was accumulated. Organizational affiliation can therefore serve as a reputational technology: the firm provides a recognizable identity under which otherwise difficult-to-evaluate agents can transact.

3.1.3 Organizationally Mediated Market Access

A third assumption is that access to customers, suppliers, capital, and other productive opportunities is substantially mediated by organizational participation.

Let A_i denote the market-access capacity of agent i. In a conventional organizational environment, it is natural to specify

$$A_i = A_i(F_i),$$

where F_i denotes the organization to which agent i belongs.

3.1.4 Scarce Coordination Capacity

A fourth assumption is that coordination capacity is scarce.

Let q denote the amount of heterogeneous productive activity that a coordinating agent or organization can effectively manage. In the canonical environment, the marginal cost of coordination generally rises with organizational scope:

$$\frac{\partial C_H}{\partial q} > 0, \qquad \frac{\partial^2 C_H}{\partial q^2} \geq 0.$$

This creates the familiar diseconomies of managerial scale.

3.1.5 Organizationally Embedded Productive Capabilities

A fifth assumption concerns the location of productive capabilities themselves. Conventional organizational theory frequently treats productive assets as either owned by firms or accessible through explicit market transactions. Human and organizational capabilities can be associated with firms through employment relationships, proprietary knowledge, routines, culture, and complementary assets.

Let k_i denote the productive capability of agent i. The canonical framework allows

$$k_i = k_i(F_i),$$

in the sense that the productivity of an agent can depend substantially on the organizational environment in which that agent operates.

3.1.6 Limited Persistent Coordination Outside the Firm

Finally, the canonical framework contains a more fundamental institutional assumption: persistent coordination among legally independent agents is either sufficiently costly to be treated as market exchange or sufficiently integrated to be treated as organization.

Let N denote a set of persistent relationships among independent agents. In the canonical formulation, one can generally represent

$$N \subseteq M$$

if network relationships are treated as contractual market exchange, or

$$N \subseteq H$$

if they involve sufficient integration to constitute organizational control.

3.2 The Canonical Boundary as a Result of These Assumptions

The preceding assumptions imply that organizational affiliation provides several economically valuable functions simultaneously. A firm can aggregate information, establish reputation, provide market access, bundle complementary capabilities, and concentrate scarce coordination capacity.

Under these conditions, the organizational problem can be represented as

$$V_M = F_M - C_M(I, \rho, A, q, k),$$

and

$$V_H = F_H - C_H(I, \rho, A, q, k).$$

The firm exists whenever

$$V_H > V_M.$$

The boundary of the firm therefore reflects the point at which the benefits of bundling these functions within a common organization cease to exceed the costs of internal administration.

The canonical framework does not require every one of these functions to be performed by the firm. Its central implication is weaker and more general: the firm becomes valuable when organizational integration reduces the total cost of coordinating productive activity.

The modern technological environment, however, increasingly permits these functions to be separated.

3.3 The Declining Cost of Information Transmission

Digital communication substantially reduces the cost of transmitting information across organizational and geographic boundaries. Let

$$\tau_I$$

denote the marginal cost of transmitting relevant information between agents.

The canonical environment can be represented by a relatively high value of τ_I. Technological progress reduces this parameter:

$$\frac{\partial \tau_I}{\partial T_I} < 0,$$

where T_I represents information and communication technology.

The direct consequence is a reduction in the coordination advantage associated with organizational proximity.

Importantly, this does not imply that information becomes costless. Tacit knowledge, contextual interpretation, and complex coordination can remain difficult. The relevant theoretical point is instead that a larger fraction of economically useful information can be transmitted without requiring common organizational membership.

As τ_I declines, the marginal cost of coordinating an activity externally falls. The Coasian boundary condition therefore changes even if every other feature of the production technology remains unchanged.

3.4 The Emergence of Portable Reputation

Digital networks introduce a qualitatively different change by altering the portability of reputation.

Let r_i denote the reputation stock accumulated by agent i. Suppose the economic value of reputation depends on the set of counterparties able to observe it. Define

$$P_i$$

as the portability of agent i's reputation across relationships.

In a conventional environment,

$$P_i \approx 0$$

for many economically relevant relationships: reputation is primarily generated through repeated bilateral interactions or organizational affiliation.

Digital networks can increase

$$P_i > 0$$

by making records of activity, demonstrated competence, endorsements, public communication, and prior performance observable to a large set of potential counterparties.

The resulting change is not simply a reduction in search costs. It changes the ownership structure of a previously organizationally mediated resource. Reputation can become an asset possessed by the individual rather than solely by the organization.

We therefore define portable network capital as

$$N_i = N(r_i, \ell_i, a_i, d_i),$$

where r_i represents reputation, ℓ_i relational capital, a_i accumulated audience or market access, and d_i distribution capacity.

The crucial property is portability:

$$\frac{\partial N_i}{\partial F_i} < 0$$

need not hold. That is, the value of N_i need not disappear when agent i leaves organization F_i.

This breaks an important source of organizational dependence.

3.5 The Unbundling of Market Access

The portability of reputation is accompanied by a decline in the degree to which market access must be mediated by firms.

Let

$$A_i = A_i(F_i, N_i),$$

where N_i represents portable network capital.

In the canonical environment,

$$\frac{\partial A_i}{\partial F_i}$$

is large relative to

$$\frac{\partial A_i}{\partial N_i}.$$

In a networked economy, the latter can become substantial. An independent agent may possess direct access to customers, collaborators, suppliers, audiences, or other markets through a network accumulated outside any particular organization.

This changes the economic meaning of organizational affiliation. Employment or formal membership no longer necessarily bundles productive capability with access to the market.

A firm can consequently lose one of its traditional sources of value even if its internal coordination technology remains unchanged.

3.6 The Decline of Matching and Search Costs

Network technologies also alter the cost of finding productive partners.

Let

$$C_S$$

denote the expected cost of locating an appropriate counterparty. In a conventional market,

$$C_S = C_S(\text{search space}, \text{information}, \text{reputation}, \text{matching technology}).$$

Improved search and recommendation systems reduce this cost:

$$\frac{\partial C_S}{\partial T_S} < 0.$$

The economic consequence is particularly important for specialized productive capabilities. A worker, entrepreneur, researcher, designer, or engineer no longer needs to locate all complementary capabilities within a single organization. A sufficiently effective matching technology can make it economical to assemble complementary capabilities externally.

The relevant unit of organization can therefore shift from the persistent firm toward the temporary or persistent network.

3.7 The Changing Cost of Coordination

Information transmission and matching technologies reduce the costs of external coordination. Artificial intelligence introduces an additional mechanism: it can directly expand the quantity and complexity of activity that a coordinating agent can manage.

Let q denote coordination capacity and let T_A denote the effectiveness of general-purpose coordination technology. We write

$$q = q(T_A), \qquad q'(T_A) > 0.$$

Equivalently, for a given amount of coordinated activity Q, let the cost of coordination be

$$C_C(Q, T_A),$$

with

$$\frac{\partial C_C}{\partial T_A} < 0.$$

Artificial intelligence is therefore potentially relevant to the theory not because it is another information technology, but because it can alter the production function of coordination itself.

3.8 From Economies of Scale to Economies of Coordination

The canonical theory generally assumes that internal coordination becomes increasingly costly as organizational scope expands:

$$C_H'(Q) > 0.$$

Suppose a general-purpose coordination technology T_A reduces this marginal cost:

$$\frac{\partial^2 C_H(Q, T_A)}{\partial Q \, \partial T_A} < 0.$$

But this effect alone does not imply smaller firms. If the same technology reduces the costs of coordinating independent agents,

$$\frac{\partial^2 C_N(Q, T_A)}{\partial Q \, \partial T_A} < 0,$$

where C_N denotes network coordination costs, then technological progress can make both hierarchical and networked organization more efficient.

3.9 The Feasible Set of Organizational Forms

Under the canonical assumptions,

$$\mathcal{S}_0 = \{M, H\}.$$

The technological environment described above permits a larger set:

$$\mathcal{S}_1 = \{M, N, H, \ldots\},$$

where N denotes distributed network coordination.

3.10 Necessary Conditions for a Distinct Network Architecture

For network coordination to constitute a genuine third architecture rather than a relabeling of market exchange, three conditions are required:

1. Agents must be able to maintain meaningful coordination relationships without surrendering residual control to a common hierarchy.

2. The network must generate complementarities that would not arise from isolated bilateral transactions.

3. The cost of sustaining these network complementarities must be sufficiently low that they generate surplus relative to both anonymous market exchange and hierarchical integration.

Let C_N denote the cost of network coordination and B_N the additional productive benefit generated by network complementarities. Network coordination is viable when

$$F_N + B_N - C_N > \max\{V_M, V_H\}.$$

3.11 A Changed Economic Primitive

We can represent the generalized coordination problem as

$$s^* = \arg\min_{s \in \mathcal{S}} C\left(s; \tau_I, P, A, T_S, T_A, K\right),$$

where τ_I is the cost of information transmission, P captures the portability of reputation and network capital, A captures independently accessible market access, T_S represents search and matching technology, T_A represents general-purpose coordination technology, and K captures the underlying productive complementarities among activities.

The canonical market–hierarchy result is recovered when portability, external market access, and independent coordination capacity are sufficiently limited and when the feasible set is restricted to

$$\mathcal{S} = \{M, H\}.$$

3.12 Implications for the Theory of the Firm

Three conclusions follow:

1. These technological changes do not overturn the logic of the canonical theory; they modify its parameters. Coase's question remains valid, but the cost of using the price mechanism is no longer independent of the technological infrastructure through which agents search, communicate, establish reputation, and coordinate.

2. The economic value of organizational integration becomes more conditional as functions migrate outside the firm. When firms no longer uniquely provide information, reputation, market access, or coordination capacity, the organizational bundle becomes partially decomposable. Some functions can migrate outside the formal boundary of the firm without destroying the productive complementarities associated with organization.

3. A decline in coordination costs can make a new institutional architecture feasible, allowing firms, markets, and networks to emerge as equilibrium outcomes of a single general coordination problem. The resulting network is not simply a cheaper market. It can possess its own coordination economies arising from persistent relationships, portable reputation, shared information, matching infrastructure, and distributed productive capabilities. This motivates the central extension of the paper.

4 Distributed Coordination and the General Organizational Problem

The preceding sections constructed the canonical market–hierarchy benchmark and identified the restrictions required for that dichotomy to characterize the organizational problem. This section relaxes those restrictions by introducing a third coordination architecture: persistent coordination among productive agents who retain independent control rights.

The objective is not to append "networks" to the canonical theory as an additional organizational form. Rather, we construct a common set of primitives from which markets, networks, and hierarchies can be represented as alternative solutions to the same coordination problem.

The central distinction is between *productive ownership* and *coordination provision.* A network permits productive agents to remain separately owned while accessing a persistent coordination infrastructure. The technology of that infrastructure is characterized by three primitives:

$$\theta = (P, A, \gamma).$$

Here P measures the portability of coordination capital across ownership boundaries, A measures the productivity and generality of coordination capacity, and γ measures the returns to scale in the production of coordination capacity itself.

The network coordination cost used in the remainder of the paper is not an additional primitive. It is induced by the equilibrium network generated by these underlying technologies.

4.1 Productive Agents

Consider a finite set of productive agents

$$\mathcal{I} = \{1, \dots, n\}.$$

Agent i possesses productive capability x_i and chooses a vector of productive actions

$$q_i \in \mathbb{R}_+^{d_i}.$$

Aggregate production is

$$Y = F(\mathbf{q}, \mathbf{x}),$$

where

$$\mathbf{q} = (q_1, \ldots, q_n), \qquad \mathbf{x} = (x_1, \ldots, x_n).$$

The production function may contain technological interdependencies across agents. In particular, the marginal productivity of an agent's action may depend on information, resources, or actions supplied by other agents.

Let

$$z \in \mathcal{Z} \subseteq \mathbb{R}_+$$

denote the coordination requirement associated with a productive activity. Higher z represents activities for which productive decisions are more strongly interdependent and therefore require greater coordination.

The organizational problem is to choose an institutional structure through which these interdependencies are coordinated.

4.2 Coordination Architectures

Let

$$\mathcal{A} = \{M, N, H\}$$

denote the set of feasible coordination architectures.

The architectures differ in the allocation of control rights and in the technology through which productive interdependencies are coordinated.

Market coordination (M). Under market coordination, productive agents retain independent control rights and coordinate through decentralized, transaction-specific exchange. Let $R_i = 1$ denote full residual control by productive agent i. Coordination relationships need not persist beyond the transactions through which they are implemented.

Hierarchy (H). Under hierarchical coordination, residual control rights are consolidated within an ownership structure. Productive activities can therefore be coordinated through authority, internal information systems, and organizational routines.

Let

$$\Pi_F$$

denote an ownership partition of the productive agents. A hierarchy permits multiple productive agents to belong to the same element of Π_F.

Network coordination (N). Under network coordination, productive agents retain independent control rights but can participate in a persistent coordination infrastructure.

Thus,

$$\boxed{N = \text{independent productive ownership} + \text{persistent coordination infrastructure.}}$$

The network does not require that the productive agents sharing coordination capacity belong to the same ownership partition.

This separation is the fundamental organizational margin introduced by the theory.

4.3 The Network as a Coordination Technology

A network is represented by a bipartite graph

$$G = (\mathcal{I}, \mathcal{J}, E),$$

where $\mathcal{I}$ is the set of productive agents, $\mathcal{J}$ is the set of potential coordination providers,

and

$$E \subseteq \mathcal{I} \times \mathcal{J}$$

is the set of active coordination relationships.

For provider $j \in \mathcal{J}$, define its participant set by

$$N_j(G) = \{i \in \mathcal{I} : (i, j) \in E\},$$

and its participant count by

$$n_j(G) = |N_j(G)|.$$

The network therefore contains two conceptually distinct objects:

$$\text{productive agents} \qquad \text{and} \qquad \text{coordination providers.}$$

A productive agent may participate in one or more coordination systems. This allows the model to accommodate single-homing or multi-homing as special cases rather than imposing either institution as a primitive.

For provider j, let

$$a_j > 0$$

denote an exogenous productivity parameter capturing the provider's underlying coordination capability.

The provider's effective coordination capacity is

$$K_j(G; A, \gamma) = A a_j n_j(G)^\gamma.$$

Aggregate network coordination capacity is therefore

$$\boxed{K_N(G; A, \gamma) = \sum_{j \in \mathcal{J}} A a_j n_j(G)^{\gamma}.}$$

This specification makes the three roles of the technological primitives distinct.

The parameter A is a common productivity and generality shifter:

$$\frac{\partial K_N}{\partial A} > 0.$$

The parameter γ determines how coordination capacity responds to the scale of a provider's participant base:

$$\frac{\partial^2 K_j}{\partial n_j^2} = A a_j \gamma(\gamma - 1) n_j^{\gamma-2}.$$

Thus,

$$\gamma > 1$$

corresponds to increasing returns to coordination scale,

$$\gamma = 1$$

to constant returns, and

$$0 < \gamma < 1$$

to decreasing returns.

Importantly, increasing returns here concern the production of *coordination capacity*, not necessarily the production of the final good.

This distinction will be central to the concentration results below.

4.4 Portable Coordination Capital

The parameter P governs the portability of coordination capital.

Let

$$\tau(P) > 0$$

denote the per-participant cost of accessing or maintaining coordination capacity across an organizational boundary, with

$$\tau'(P) < 0.$$

Thus greater portability lowers the cost of establishing and maintaining persistent coordination relationships without requiring ownership integration.

The parameter P can therefore represent, in reduced form, improvements in technologies or institutions that allow relationships, information, reputation, audience access, communication channels, interaction histories, or other coordination assets to remain productive across organizational boundaries.

The defining property is not that P raises output directly, but that it changes the cost of making coordination capacity available to independently owned productive agents:

$$\boxed{P\uparrow \implies \tau(P)\downarrow.}$$

This distinction separates portable coordination capital from a conventional productivity shifter.

A technology can raise the productivity of production while leaving organizational boundaries unchanged. Such a technology need not increase P.

4.5 The Induced Cost of Network Coordination

To connect the network technology to the organizational problem, let

$$\phi : \mathbb{R}_+ \to \mathbb{R}_+$$

denote the residual cost of satisfying coordination requirements. We assume

$$\phi'(x) > 0, \qquad \phi''(x) \geq 0.$$

A network with greater effective coordination capacity therefore requires less resource expenditure to satisfy a given coordination requirement.

For a network G, define the induced coordination cost as

$$\mathcal{C}_N(z, G; \theta) = \phi\left(\frac{z}{K_N(G; A, \gamma)}\right) + \tau(P)L(G) + \sum_{j \in \mathcal{J}} f_j \mathbf{1}\{n_j(G) > 0\},$$

where

$$L(G) = \sum_{j \in \mathcal{J}} n_j(G)$$

is the number of active participant-provider relationships and $f_j \geq 0$ is a provider-specific fixed operating cost.

The first term captures the residual cost of meeting coordination requirements given the capacity supplied by the network.

The second term captures the cost of maintaining persistent cross-boundary relationships.

The final term captures the fixed cost of activating a coordination provider.

This formulation is deliberately general enough to accommodate different interpretations of coordination infrastructure while imposing the crucial structural restrictions

$$\frac{\partial \tau}{\partial P} < 0, \qquad \frac{\partial K_N}{\partial A} > 0.$$

It follows directly that, holding the network fixed,

$$\frac{\partial \mathcal{C}_N}{\partial P} < 0$$

and

$$\frac{\partial \mathcal{C}_N}{\partial A} < 0.$$

The effect of γ operates through the scale of coordination capacity and is therefore endogenous to the network structure.

4.6 Network Formation

The equilibrium network is chosen by the agents and coordination providers subject to the technology above.

Let

$$W(G; z, \theta)$$

denote aggregate surplus generated by network G, net of coordination costs and provider activation costs:

$$W(G; z, \theta) = V(z, G) - \mathcal{C}_N(z, G; \theta),$$

where $V(z, G)$ denotes the gross productive surplus generated when the coordination requirements of the productive system are satisfied through G.

The exact allocation of surplus among productive agents and coordination providers is not needed for the aggregate organizational problem, but it matters for individual link formation. Section 7 therefore specifies the corresponding decentralized network-formation game and establishes the conditions under which its equilibrium coincides with the efficient network for the class of environments considered here.

For the aggregate problem, define the set of technologically and privately feasible networks by

$$\mathcal{G}(\theta).$$

The equilibrium coordination network is denoted by

$$G^*(z;\theta) \in \arg\max_{G\in\mathcal{G}(\theta)} W(G;z,\theta),$$

when decentralized incentives implement the efficient network. More generally, G^* denotes the equilibrium selected by the network-formation game developed in Section 7.

The equilibrium network coordination cost is consequently

$$\boxed{C_N^*(z;\theta) = \mathcal{C}_N\left(z, G^*(z;\theta);\theta\right).}$$

This definition is important: C_N^* is an *induced* object.

It incorporates both

direct technological effects

and

endogenous network-formation effects.

Hence,

$$\frac{dC_N^*}{dP} = \frac{\partial \mathcal{C}_N}{\partial P} + \frac{\partial \mathcal{C}_N}{\partial G}\frac{\partial G^*}{\partial P},$$

with an analogous expression for A and γ.

This decomposition will allow the paper to distinguish the direct effect of technological progress from the organizational response it induces.

4.7 Network Scale as an Aggregate State Variable

Although the general network is represented by the graph G, some of the organizational-boundary results depend only on aggregate network scale.

Define

$$m(G) = \sum_{j \in \mathcal{J}} n_j(G),$$

and let

$$m^*(z; \theta) = m(G^*(z; \theta)).$$

We impose the following aggregation condition for the main-text comparative statics.

Assumption 1 (Symmetric network aggregation)**.** *Conditional on network scale m, productive agents are exchangeable and the distribution of coordination costs is symmetric. There exists an aggregate capacity function*

$$\overline{K}_N(m; A, \gamma)$$

such that all networks equivalent under the symmetry restriction generate the same aggregate coordination capacity.

The assumption does not imply that topology is irrelevant.

Rather, it states that topology can be treated explicitly in the network formation problem while the principal organizational-boundary results can be expressed in terms of the sufficient statistic m.

Under this aggregation,

$$K_N(G; A, \gamma) = \overline{K}_N(m(G); A, \gamma),$$

and therefore

$$C_N^*(z; \theta) = C_N(z; m^*(z; \theta), \theta).$$

The full graph remains the underlying object of the theory; m is a reduced-form state variable used when topology does not independently affect the organizational comparison.

4.8 The Organizational Problem

Let the equilibrium coordination cost under the market architecture be

$$C_M^*(z),$$

and the equilibrium coordination cost under hierarchy be

$$C_H^*(z).$$

These objects incorporate the transaction, information, authority, incentive, and internal-coordination mechanisms developed in the canonical benchmark.

For the network architecture, the relevant cost is the induced equilibrium cost

$$C_N^*(z; \theta).$$

The organizational problem is therefore

$$\boxed{C^*(z; \theta) = \min \left\{ C_M^*(z), C_N^*(z; \theta), C_H^*(z) \right\}.}$$

The equilibrium organizational architecture is

$$\sigma^*(z;\theta) \in \arg\min_{\sigma\in\{M,N,H\}} C^*_\sigma(z;\theta).$$

The canonical market–hierarchy model is recovered as the special case in which network coordination is either unavailable or never strictly dominates the two incumbent architectures:

$$\mathcal{G}(\theta) = \varnothing$$

or

$$C^*_N(z;\theta) \geq \min\{C^*_M(z), C^*_H(z)\}$$

for all relevant z.

The new theory therefore does not discard the canonical theory.

It nests it.

4.9 Ownership and Coordination Boundaries

The introduction of a persistent network requires distinguishing two organizational boundaries.

Let

$$\Pi_F$$

denote the ownership partition of productive agents. For agent i, define its ownership component by

$$B_F(i;\Pi_F) = \{k \in \mathcal{I} : k \text{ belongs to the same ownership unit as } i\}.$$

Let

$$G^*$$

be the equilibrium coordination network. Define the effective coordination component of agent i as

$$B_C(i; G^*) = \{k \in \mathcal{I} : i \text{ and } k \text{ are connected through the equilibrium coordination structure}\}.$$

The two boundaries coincide under the canonical hierarchical benchmark when all economically relevant coordination occurs within ownership units:

$$B_C(i; G^*) = B_F(i; \Pi_F).$$

Under network organization, however, it is possible that

$$B_C(i; G^*) \supsetneq B_F(i; \Pi_F).$$

Define the organizational-boundary gap for agent i by

$$\Delta_B(i) = \mu \left(B_C(i; G^*) \setminus B_F(i; \Pi_F) \right),$$

where μ is a measure of productive activity.

An aggregate boundary gap is

$$\Delta_B = \sum_{i \in \mathcal{I}} \omega_i \Delta_B(i), \qquad \omega_i \geq 0, \qquad \sum_i \omega_i = 1.$$

This quantity measures the extent to which productive activity can be economically integrated through coordination without corresponding ownership integration.

The central organizational possibility of the theory can therefore be written

$$\boxed{\Delta_B > 0.}$$

Technological progress in portability or general-purpose coordination capacity can enlarge this gap by lowering the cost of coordinating agents who remain outside the same ownership structure.

4.10 Ownership Concentration and Coordination Concentration

The distinction between ownership and coordination boundaries implies a corresponding distinction between concentration in productive ownership and concentration in coordination provision.

Let

$$\mathbf{s} = (s_1, \ldots, s_n), \qquad \sum_i s_i = 1,$$

denote productive ownership shares. Define the ownership concentration index

$$\boxed{H_F = \sum_{i=1}^{n} s_i^2.}$$

For coordination providers, let

$$K_j^* = K_j(G^*; A, \gamma)$$

and define coordination-capacity shares

$$r_j = \frac{K_j^*}{\sum_{k \in \mathcal{J}} K_k^*}, \qquad \sum_j r_j = 1.$$

Coordination concentration is

$$\boxed{H_C = \sum_{j \in \mathcal{J}} r_j^2.}$$

The two indices measure distinct economic objects.

Nothing in the model requires

$$H_F = H_C.$$

Indeed, a central prediction of the theory is that technological progress can produce

$$H_F \downarrow$$

while simultaneously producing

$$H_C \uparrow.$$

The possibility arises because productive ownership and coordination provision are governed by different equilibrium margins.

4.11 Structural Interpretation of the Technology Parameters

The preceding construction gives a unified interpretation of the structural parameter vector

$$\boxed{\theta = (P, A, \gamma).}$$

The three parameters operate through distinct channels.

First,

$$P$$

governs the portability of coordination capital and therefore the cost of maintaining persistent coordination across ownership boundaries.

Second,

$$A$$

governs the productivity and generality of coordination capacity once the coordination infrastructure is deployed.

Third,

$$\gamma$$

governs the elasticity of coordination capacity with respect to provider scale.

Thus,

$$P \to \text{access cost},$$

$$A \to \text{coordination capacity},$$

and

$$\gamma \to \text{scale elasticity}.$$

These are distinct technological margins.

A communication technology may primarily increase P.

A general-purpose artificial intelligence system may primarily increase A.

A technology that allows the same coordination infrastructure to serve many more heterogeneous agents may increase γ.

The theory therefore does not require social media or artificial intelligence to be treated as primitive categories. They are potential technologies that change the underlying coordination parameters.

4.12 Equilibrium Objects

An equilibrium of the model consists of an organizational architecture

$$\sigma^*,$$

an ownership partition

$$\Pi_F^*,$$

and, whenever

$$\sigma^* = N,$$

an equilibrium coordination network

$$G^*,$$

such that:

1. productive agents optimize conditional on the available coordination architecture;
2. the equilibrium network satisfies the network-formation conditions specified in Section 7;
3. ownership assignments are optimal given the coordination technology;
4. the selected architecture minimizes the equilibrium coordination cost among the feasible alternatives.

We may therefore write the equilibrium correspondence as

$$\boxed{(\Pi_F^*, G^*, \sigma^*) = \mathcal{E}(z, \theta).}$$

When the equilibrium is locally unique, this correspondence can be represented as a function.

The important point is that organizational form is now endogenous to the coordination technology:

$$\boxed{(P, A, \gamma) \to G^* \to C_N^* \to \sigma^*.}$$

This mapping is the organizing structure for the remainder of the paper.

Section 5 derives the comparative statics of the organizational problem conditional on the equilibrium network. Section 7 then derives the network equilibrium itself and establishes the conditions under which increasing returns generate coordination concentration and organizational-boundary decoupling.

5 Comparative Statics of Organizational Choice

Section 4 established the common organizational problem and derived the equilibrium cost of network coordination from an underlying coordination technology. This section characterizes the comparative statics of organizational choice without yet imposing the full network-formation structure developed in Section 7.

The distinction is important. The effects derived here arise from changes in the technological environment conditional on the induced network. Section 7 subsequently determines how the network itself responds to those changes.

The principal result is that improvements in portable and general-purpose coordination technology can expand the set of activities for which persistent network coordination is cost-minimizing. The resulting organizational boundary is therefore endogenous to the technology of coordination rather than fixed by the distinction between “market” and “firm.”

5.1 Direct Effects of Coordination Technology

Recall that the equilibrium network coordination cost is

$$C_N^*(z;\theta) = \mathcal{C}_N\left(z, G^*(z;\theta);\theta\right),$$

where

$$\theta = (P, A, \gamma).$$

For the comparative statics in this section, hold the equilibrium network G^* fixed. The resulting derivatives isolate the *direct technological effects* of changes in the coordination environment.

From equation (4.5),

$$\mathcal{C}_N(z, G;\theta) = \phi\left(\frac{z}{K_N(G; A, \gamma)}\right) + \tau(P)L(G) + \sum_{j\in\mathcal{J}} f_j \mathbf{1}\{n_j(G) > 0\}.$$

Since

$$\tau'(P) < 0,$$

we obtain

$$\frac{\partial \mathcal{C}_N}{\partial P} = \tau'(P)L(G) < 0.$$

Thus, holding the network fixed,

$$\boxed{P\uparrow \implies C_N\downarrow.}$$

The economic interpretation is straightforward. Greater portability reduces the cost of maintaining coordination relationships across ownership boundaries. It therefore improves the relative efficiency of a network without requiring ownership integration.

Likewise,

$$K_N(G; A, \gamma) = A \sum_{j \in \mathcal{J}} a_j n_j(G)^\gamma,$$

so

$$\frac{\partial K_N}{\partial A} = \sum_{j \in \mathcal{J}} a_j n_j^\gamma > 0.$$

Since $\phi' > 0$,

$$\frac{\partial \mathcal{C}_N}{\partial A} = \phi' \left(\frac{z}{K_N} \right) \left(-\frac{z}{K_N^2} \right) \frac{\partial K_N}{\partial A} < 0.$$

Hence,

$$\boxed{A \uparrow \quad \Longrightarrow \quad C_N \downarrow.}$$

The parameter A therefore shifts the productivity of the coordination infrastructure itself.

The effect of γ is different. Holding G fixed,

$$\frac{\partial K_N}{\partial \gamma} = A \sum_{j \in \mathcal{J}} a_j n_j^\gamma \log n_j.$$

Whenever the relevant providers have

$$n_j > 1,$$

we have

$$\frac{\partial K_N}{\partial \gamma} > 0,$$

and consequently

$$\frac{\partial \mathcal{C}_N}{\partial \gamma} < 0.$$

Thus,

$$\boxed{\gamma \uparrow \quad \Longrightarrow \quad C_N \downarrow}$$

for networks with nontrivial provider scale.

The effect is scale-dependent: increasing γ has little effect on coordination systems serving only one participant, but becomes increasingly important as coordination providers serve larger populations.

5.2 The Envelope Decomposition

The preceding derivatives hold the network fixed. In equilibrium, however, changes in (P, A, γ) also change the network itself.

Let

$$G^*(z; \theta)$$

denote the equilibrium network. The total derivative of equilibrium network coordination cost with respect to any scalar technological parameter $\theta_k \in \{P, A, \gamma\}$ is

$$\frac{dC_N^*}{d\theta_k} = \frac{\partial \mathcal{C}_N}{\partial \theta_k} + D_G \mathcal{C}_N \left[\frac{\partial G^*}{\partial \theta_k} \right].$$

The first term is the direct technological effect. The second is the *network-response effect.*

This decomposition is central to the theory.

For P,

$$\frac{dC_N^*}{dP} = \tau'(P)L(G^*) + D_G\mathcal{C}_N\left[\frac{\partial G^*}{\partial P}\right].$$

For A,

$$\frac{dC_N^*}{dA} = \frac{\partial \mathcal{C}_N}{\partial A} + D_G\mathcal{C}_N\left[\frac{\partial G^*}{\partial A}\right].$$

For γ,

$$\frac{dC_N^*}{d\gamma} = \frac{\partial \mathcal{C}_N}{\partial \gamma} + D_G\mathcal{C}_N\left[\frac{\partial G^*}{\partial \gamma}\right].$$

If the network is chosen to minimize the coordination cost itself, the network-response term vanishes under the standard envelope conditions. If network formation is decentralized and generates an equilibrium that is not globally cost-minimizing, the term generally does not vanish.

This distinction will matter in Section 7, where the network-formation equilibrium is derived explicitly.

5.3 The Network Region

The organizational choice problem is

$$C^*(z;\theta) = \min\left\{C_M^*(z), C_N^*(z;\theta), C_H^*(z)\right\}.$$

Define the market–network boundary by

$$\mathcal{B}_{MN} = \left\{(z,\theta) : C_M^*(z) = C_N^*(z;\theta) \leq C_H^*(z)\right\},$$

and the network–hierarchy boundary by

$$\mathcal{B}_{NH} = \{(z, \theta) : C_N^*(z; \theta) = C_H^*(z) \le C_M^*(z)\}.$$

The set of activities for which network coordination is optimal is

$$\boxed{\Omega_N(\theta) = \{z : C_N^*(z; \theta) < \min\{C_M^*(z), C_H^*(z)\}\}.}$$

The theory therefore predicts a region of the activity space in which persistent network coordination dominates both decentralized market exchange and hierarchical integration.

The region is empty when network coordination is never cost-minimizing and expands when technological progress lowers network coordination costs relative to the two incumbent architectures.

5.4 Market–Network Boundary

Define

$$D_{MN}(z; \theta) = C_N^*(z; \theta) - C_M^*(z).$$

The market–network boundary satisfies

$$D_{MN}(z; \theta) = 0.$$

Suppose that at an interior boundary point $z_{MN}(\theta)$,

$$\frac{\partial D_{MN}}{\partial z} \neq 0.$$

The implicit function theorem therefore gives

$$\frac{dz_{MN}}{d\theta_k} = -\frac{\partial D_{MN}/\partial \theta_k}{\partial D_{MN}/\partial z}.$$

Suppose further that greater coordination requirements increase the relative cost advantage of network coordination over decentralized exchange, so that

$$\frac{\partial D_{MN}}{\partial z} > 0.$$

Then, for P,

$$\frac{\partial D_{MN}}{\partial P} = \frac{dC_N^*}{dP} < 0$$

whenever the total effect of portability is cost-reducing.

It follows that

$$\boxed{\frac{dz_{MN}}{dP} > 0.}$$

Thus, greater portability expands the set of activities for which network coordination dominates market exchange.

Analogously, if

$$\frac{dC_N^*}{dA} < 0$$

and

$$\frac{\partial D_{MN}}{\partial z} > 0,$$

then

$$\boxed{\frac{dz_{MN}}{dA} > 0.}$$

The same logic applies to γ whenever

$$\frac{dC_N^*}{d\gamma} < 0.$$

5.5 Network–Hierarchy Boundary

Define

$$D_{NH}(z;\theta) = C_H^*(z) - C_N^*(z;\theta).$$

The network–hierarchy boundary satisfies

$$D_{NH}(z;\theta) = 0.$$

Suppose that at an interior boundary point

$z_{NH}(\theta)$,

$$\frac{\partial D_{NH}}{\partial z} \neq 0.$$

Then

$$\frac{dz_{NH}}{d\theta_k} = -\frac{\partial D_{NH}/\partial \theta_k}{\partial D_{NH}/\partial z}.$$

Suppose that higher coordination requirements eventually make hierarchical integration relatively more attractive, so that

$$\frac{\partial D_{NH}}{\partial z} < 0.$$

If portability reduces network coordination cost,

$$\frac{dC_N^*}{dP} < 0,$$

then

$$\frac{\partial D_{NH}}{\partial P} = -\frac{dC_N^*}{dP} > 0.$$

Consequently,

$$\boxed{\frac{dz_{NH}}{dP} > 0.}$$

The interpretation is that portability permits network coordination to remain competitive with hierarchy over a larger range of coordination requirements.

Likewise, whenever

$$\frac{dC_N^*}{dA} < 0,$$

we obtain

$$\boxed{\frac{dz_{NH}}{dA} > 0.}$$

Under the analogous condition for γ,

$$\boxed{\frac{dz_{NH}}{d\gamma} > 0.}$$

5.6 Expansion of the Network Region

The preceding results can be combined into a simple sufficient condition for the expansion of the network region.

Proposition 1 (Expansion of the network region)**.** *Suppose that:*

1. *$C_M^*(z)$ and $C_H^*(z)$ do not depend directly on P or A;*

2. the equilibrium network cost satisfies

$$\frac{dC_N^*}{dP} < 0, \qquad \frac{dC_N^*}{dA} < 0;$$

3. the market–network and network–hierarchy boundaries are interior and satisfy

$$\frac{\partial D_{MN}}{\partial z} > 0, \qquad \frac{\partial D_{NH}}{\partial z} < 0.$$

Then an increase in either P or A weakly expands the set of coordination requirements for which network coordination is cost-minimizing. If both boundaries remain interior, the network region expands strictly whenever the corresponding derivative of C_N^* is strictly negative.

Proof. The market–network boundary is defined by

$$D_{MN}(z;\theta) = 0.$$

By the implicit function theorem,

$$\frac{dz_{MN}}{d\theta_k} = -\frac{\partial D_{MN}/\partial\theta_k}{\partial D_{MN}/\partial z}.$$

For $\theta_k = P$,

$$\frac{\partial D_{MN}}{\partial P} = \frac{dC_N^*}{dP} < 0.$$

Since

$$\frac{\partial D_{MN}}{\partial z} > 0,$$

we obtain

$$\frac{dz_{MN}}{dP} > 0.$$

The same argument gives

$$\frac{dz_{MN}}{dA} > 0.$$

For the network–hierarchy boundary,

$$D_{NH}(z;\theta) = 0,$$

and therefore

$$\frac{dz_{NH}}{d\theta_k} = -\frac{\partial D_{NH}/\partial\theta_k}{\partial D_{NH}/\partial z}.$$

Since

$$\frac{\partial D_{NH}}{\partial P} = -\frac{dC_N^*}{dP} > 0$$

and

$$\partial D_{NH}/\partial z < 0$$

,

we obtain

$$\frac{dz_{NH}}{dP} > 0.$$

Analogously,

$$\frac{dz_{NH}}{dA} > 0.$$

Thus the interval of z values for which

$$C_N^*(z;\theta) < \min\{C_M^*(z), C_H^*(z)\}$$

expands weakly with P and A, and strictly when the relevant cost effect is strict. $\square$

5.7 General-Purpose Coordination and Organizational Scale

The parameter A differs from a conventional sector-specific productivity parameter because it increases the effectiveness of coordination capacity across a broad set of productive activities.

Suppose the coordination requirement z varies across activities according to a distribution

$$F_Z(z).$$

Let

$$\Omega_N(\theta)$$

denote the network region defined in equation (5.3). The aggregate share of activities coordinated through networks is

$$S_N(\theta) = \int_{\Omega_N(\theta)} dF_Z(z).$$

Under the conditions of Proposition 1,

$$\frac{\partial S_N}{\partial P} \geq 0, \qquad \frac{\partial S_N}{\partial A} \geq 0.$$

If the distribution of coordination requirements assigns positive probability to the margins at which the organizational boundaries move, then the inequalities are strict:

$$\boxed{\frac{\partial S_N}{\partial P} > 0, \qquad \frac{\partial S_N}{\partial A} > 0.}$$

Thus improvements in portable or general-purpose coordination technology can increase the equilibrium prevalence of network organization.

This result does not imply that firms become uniformly smaller.

The direction of organizational scale depends on which boundary is being considered. A network can replace a hierarchical boundary while simultaneously creating a larger effective coordination system.

Consequently, the theory distinguishes:

$$\text{ownership scale}$$

from

$$\text{coordination scale.}$$

5.8 Organizational Scale and Concentration

Let

$$S_F$$

denote the average scale of an ownership unit and

$$S_C$$

the average scale of the coordination system to which a productive agent has access.

The theory places no general sign restriction on

$$\frac{\partial S_F}{\partial P}, \qquad \frac{\partial S_C}{\partial P}.$$

Indeed, the defining possibility of network organization is

$$\frac{\partial S_F}{\partial P} < 0$$

together with

$$\frac{\partial S_C}{\partial P} > 0.$$

This is not a contradiction. A decline in ownership scale can occur because productive agents no longer need to be jointly owned in order to access the coordination capacity required for production.

Similarly, general-purpose coordination capital can increase the effective scale of the coordination system while leaving the ownership structure fragmented.

The model therefore rejects the presumption that technological progress has a single organizational-scale effect.

Instead,

$$\boxed{\text{technological progress can expand coordination scale while reducing ownership scale.}}$$

Whether this occurs in equilibrium depends on the network-formation and ownership responses derived in Section 7.

5.9 The Role of Increasing Returns

The parameter γ deserves separate treatment because it affects the network not only through the level of coordination capacity but through its response to scale.

For provider j,

$$K_j = A a_j n_j^{\gamma}.$$

The elasticity of coordination capacity with respect to participant scale is

$$\frac{\partial \log K_j}{\partial \log n_j} = \gamma.$$

Hence γ has a direct interpretation as the scale elasticity of coordination provision.

When

$$\gamma < 1,$$

coordination capacity exhibits decreasing returns to provider scale.

When

$$\gamma = 1,$$

coordination capacity is proportional to provider scale.

When

$$\gamma > 1,$$

coordination capacity exhibits increasing returns.

The latter case creates the possibility that larger coordination providers become disproportionately productive as participation expands.

However, increasing returns alone do not imply that equilibrium coordination concentration must increase.

That conclusion additionally requires an equilibrium mechanism through which participants are allocated across providers and provider-level productivity differences affect those allocations.

This distinction is deliberate. Section 7 derives that mechanism explicitly.

5.10 A First Organizational-Boundary Result

The preceding analysis yields a preliminary boundary result.

Proposition 2 (Technological expansion of network organization)**.** *Suppose the market and hierarchical coordination costs are invariant to P, A, and γ, and suppose the equilibrium network cost satisfies*

$$\frac{dC_N^*}{dP} < 0, \qquad \frac{dC_N^*}{dA} < 0.$$

Suppose further that the market–network and network–hierarchy boundaries are regular interior solutions. Then increases in P or A expand the set of coordination requirements for which network coordination is optimal.

If, in addition,

$$\frac{dC_N^*}{d\gamma} < 0,$$

then an increase in γ also expands the network region.

Proof. Immediate from Proposition 1 and the implicit-function characterization of the two organizational boundaries. □

This proposition establishes the first comparative-static implication of the new theory:

improvements in the technology of distributed coordination can move activities from markets or hierarch

The proposition is intentionally weaker than a claim about concentration or ownership structure. Those outcomes depend on the endogenous structure of the network itself.

5.11 Limits of the Comparative Statics

Three qualifications are important.

First, the sign of the total effect of a technological parameter on network coordination cost need not equal its direct effect. A technological improvement can induce changes in network topology that alter coordination costs in either direction.

Second, the expansion of the network region does not imply that networks become more concentrated. Network entry, multi-homing, heterogeneous provider productivity, and increasing returns can generate different concentration responses.

Third, nothing in the comparative statics implies that network organization dominates hierarchy at all levels of coordination intensity. If hierarchical coordination has sufficiently strong advantages at high levels of interdependence, the network region can remain bounded:

$$z_{MN} < z_{NH}.$$

The resulting organizational structure is therefore potentially ordered as

$$M \longrightarrow N \longrightarrow H$$

as coordination requirements increase.

This ordering is not imposed by the model. It emerges when the corresponding cost functions satisfy the regularity conditions above.

The possibility of a non-monotone ordering, multiple organizational transitions, or discontinuous changes is left to the network-formation analysis.

5.12 Summary

The main comparative-static results can be summarized as follows.

1. Greater portability P lowers the direct cost of persistent cross-boundary coordination.
2. Greater general-purpose coordination productivity A increases the capacity of a given network to satisfy coordination requirements.
3. Greater scale elasticity γ increases the productivity of sufficiently large coordination systems.
4. Consequently, under regularity conditions, increases in P and A expand the region in which network coordination is preferred to both decentralized market exchange and hierarchical integration.
5. These results concern organizational boundaries and coordination scale. They do not by themselves determine the concentration of coordination provision.

The last distinction is essential. The network can become more economically important without becoming more concentrated, and increasing returns can generate concentration only when they interact with an equilibrium allocation mechanism.

Section 7 supplies that missing mechanism. It characterizes equilibrium network formation, derives the conditions under which increasing returns generate provider concentration, and establishes the relationship between coordination concentration and the ownership boundary.

6 Relation to Existing Theories of the Firm

The framework developed above is intended as a general theory of organizational choice rather than as a theory specific to any particular technology. Its novelty lies in introducing an endogenous coordination infrastructure that can extend beyond the boundaries of productive ownership. This section relates the framework to the principal traditions in the theory of the firm, organizational economics, network economics, platforms, and technological change **?**Williamson (1975, 1985); Jackson (2008); Rochet and Tirole (2003).

The objective is not to collapse these traditions into a single mechanism. Rather, the framework identifies the common organizational problem to which their principal insights correspond and then introduces a margin that is absent, or not central, in the canonical formulation: the independent provision of persistent coordination capacity.

6.1 Transaction-Cost Economics

The transaction-cost tradition explains organizational boundaries by comparing the costs of coordinating economic activity through markets with the costs of coordinating the same activity within an organization **?**Williamson (1975, 1985).

In the present framework, this logic is represented by the comparison between

$$C_M^*(z) \quad \text{and} \quad C_H^*(z).$$

Market exchange is preferred when

$$C_M^*(z) < C_H^*(z),$$

while hierarchical organization is preferred when

$$C_H^*(z) < C_M^*(z).$$

The canonical market–hierarchy boundary is therefore a special case of the general organizational problem

$$C^*(z;\theta) = \min_{\sigma\in\{M,N,H\}} C_\sigma^*(z;\theta).$$

A persistent network can reduce coordination costs through

$$C_N^*(z;\theta) < \min\{C_M^*(z), C_H^*(z)\},$$

without requiring productive agents to surrender independent control rights.

The relevant comparison is therefore

$$\boxed{M \quad \text{vs.} \quad N \quad \text{vs.} \quad H.}$$

—

6.2 Team Production and Coordination

Team-production theories emphasize that production may require the contributions of multiple agents whose actions cannot be perfectly separated or monitored Alchian and Demsetz (1972); Holmstrom (1982).

Let

$$q = (q_1, \dots, q_n)$$

denote the vector of productive actions. The resulting output

$$F(q)$$

may depend on interactions among agents.

Under network organization, coordination can be implemented through a persistent infrastructure whose capacity is

$$K_N(G; A, \gamma) = A \sum_j a_j n_j(G)^\gamma.$$

This distinction allows team production and network coordination to coexist without conflating them: team production describes the technology, while network organization describes the coordination mechanism.

—

6.3 Property Rights and the Allocation of Control

The property-rights tradition emphasizes ownership and residual control rights when contracts are incomplete Grossman and Hart (1986); Hart and Moore (1990).

Let R_i denote the residual control rights allocated to agent i, and let Π_F represent the ownership partition. The canonical property-rights environment associates ownership with coordination:

$$B_C(i; G^*) \approx B_F(i; \Pi_F).$$

The network environment permits

$$B_C(i; G^*) \supsetneq B_F(i; \Pi_F),$$

creating a boundary gap

$$\Delta_B(i) = \mu\big(B_C(i; G^*) \setminus B_F(i; \Pi_F)\big).$$

Thus, ownership determines control rights, while coordination technology determines how far interdependence can extend beyond ownership.

—

6.4 Organization Design

Organization-design models study how authority, information, incentives, and interdependencies jointly determine structure Milgrom and Roberts (1990); Rajan and Wulf (2006); Aghion and Tirole (1997).

Here, the decision problem additionally includes whether the coordination infrastructure itself should be internalized. Let

$$\mathcal{C}_M, \quad \mathcal{C}_N, \quad \mathcal{C}_H$$

denote the costs of the three architectures. The network architecture introduces a separable coordination technology $K_N(G; A, \gamma)$, allowing coordination capacity to be accessed without acquiring productive agents.

—

6.5 Networks and Relational Organization

The network-economics literature studies how outcomes depend on relationship structure Jackson and Wolinsky (1996); Jackson (2008). Here, the network is itself an organizational technology:

$$G^*(z; \theta),$$

chosen in response to coordination requirements.

—

6.6 Platforms

Platforms are a special case of network coordination Rochet and Tirole (2003); Armstrong (2006). A platform corresponds to coordination providers $j \in \mathcal{J}$ with participant sets $N_j(G)$, and capacity

$$K_j = A a_j n_j^{\gamma}.$$

Thus,

$$\text{platform} \subseteq \text{network coordination architecture.}$$

—

6.7 Technological Change and the Firm

Technological change alters coordination costs independently of production technology. The structural vector

$$\theta = (P, A, \gamma)$$

captures portability, productivity, and scale elasticity of coordination.

—

6.8 Social Media as Coordination Technology

Social media increases portability P and potentially scale elasticity γ, allowing decentralized agents to participate in large coordination systems.

—

6.9 Artificial Intelligence as Coordination Capital

AI increases coordination productivity (A) and may increase scale elasticity (γ). Distinguishing these effects separates general-purpose productivity gains from scale-concentration effects.

—

6.10 Framework Contribution

The framework introduces a common margin:

$$\boxed{\text{coordination provision}}$$

separating ownership boundaries from coordination boundaries:

$$\Pi_F \neq \Pi(G^*).$$

—

6.11 Summary

Mapping to traditions:

$$\begin{array}{lcl} \text{Transaction costs} & \to & C_M^*, C_H^* \\ \text{Team production} & \to & z \\ \text{Property rights} & \to & \Pi_F \\ \text{Organization design} & \to & \text{control + coordination} \\ \text{Network economics} & \to & G^* \\ \text{Platforms} & \to & j \\ \text{Technological change} & \to & (P, A, \gamma) \end{array}$$

Distinctive implication:

$$\boxed{\text{ownership boundary} \neq \text{coordination boundary}}$$

The next section makes this distinction operational by solving the endogenous network-formation problem and determining when the technology of coordination generates concentration in coordination provision even as productive ownership becomes more decentralized.

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7 Endogenous Network Formation and the Organizational Boundary

Sections 4–5 treated the network architecture as an organizational technology and derived the comparative statics of organizational choice conditional on its equilibrium cost. Section 6 showed that the resulting framework extends the canonical market–hierarchy problem by separating the boundary of productive ownership from the boundary of economic coordination.

This section closes the model by making the coordination network endogenous. Agents choose whether and through whom to participate in persistent coordination relationships. The resulting equilibrium determines both the network architecture and the organizational boundary.

The analysis yields three results. First, under standard regularity conditions, the network-formation problem admits a stable equilibrium characterized by sorting toward more productive coordination providers when scale economies are sufficiently strong. Second, the resulting network determines an endogenous boundary between market exchange, network coordination, and hierarchy. Third, and most importantly, the concentration of coordination provision need not coincide with the concentration of productive ownership. Under a transparent set of conditions, technological improvements can therefore increase coordination concentration while reducing ownership concentration.

7.1 The Network-Formation Environment

Consider a continuum of productive agents indexed by $i \in [0, 1]$. Agent i has productive type

$$\omega_i \in \Omega \subseteq \mathbb{R}_+,$$

where ω_i captures productivity, specialization, or the value of participating in a coordination system.

There is a finite set of potential coordination providers

$$\mathcal{J} = \{1, \ldots, J\}.$$

Provider j has coordination productivity

$$a_j > 0.$$

An allocation is a measurable assignment

$$g : [0, 1] \to \mathcal{J} \cup \{0\},$$

where $g(i) = j$ means that agent i participates in provider j, while $g(i) = 0$ denotes decentralized market coordination.

Let

$$n_j(g) = \mu\{i : g(i) = j\}$$

denote the measure of agents assigned to provider j.

The coordination capacity of provider j is

$$K_j(g; A, \gamma) = A a_j n_j(g)^\gamma.$$

This specification preserves the distinction established in Section 5: A shifts the productivity of coordination capacity, while γ determines its elasticity with respect to the scale of participation.

For an agent of type ω, the gross benefit from participating in provider j is

$$B(\omega, K_j),$$

where

$$B_K > 0.$$

The agent's participation cost is

$$c_j(\omega),$$

which may incorporate access costs, compatibility costs, switching costs, or provider-specific contractual terms.

The net payoff from provider j is therefore

$$u_j(\omega; g) = B(\omega, K_j(g; A, \gamma)) - c_j(\omega).$$

The market alternative yields

$$u_M(\omega).$$

Agents choose the coordination architecture that maximizes net payoff.

7.2 Equilibrium Network Formation

An allocation g^* constitutes a network-formation equilibrium if, for almost every agent i,

$$g^*(i) \in \arg \max_{j \in \mathcal{J} \cup \{0\}} u_j(\omega_i; g^*).$$

The equilibrium is therefore a fixed point: provider participation determines provider scale, provider scale determines coordination capacity, and coordination capacity determines participation.

This feedback is the source of endogenous network effects.

Define the participation advantage of provider j over the market as

$$\Delta_j(\omega; g) = B(\omega, Aa_j n_j(g)^\gamma) - c_j(\omega) - u_M(\omega).$$

An agent participates in provider j whenever

$$\Delta_j(\omega; g) \geq 0$$

and provider j yields the highest payoff among available alternatives.

7.3 Existence

The following result provides the basic existence condition.

Proposition 3 (Existence of network-formation equilibrium)**.** *Suppose that Ω is compact, $B(\omega, K)$ is continuous in both arguments, $B_K > 0$, participation costs are continuous, and the induced best-response correspondence is nonempty and upper hemicontinuous. Then a network-formation equilibrium exists.*

Proof. The set of feasible participation allocations is compact under the weak topology on measurable assignments. Continuity of B and of participation costs implies that the induced payoff correspondence is upper hemicontinuous. By the standard fixed-point theorem for compact, convex-valued correspondences, a fixed point of the best-response correspondence exists. $\square$

The proposition is deliberately stated at a high level. The economically important results concern the structure of equilibrium rather than existence alone.

7.4 Sorting and Provider Scale

Suppose that provider j is more productive than provider k:

$$a_j > a_k.$$

The scale effect generated by γ determines whether this productivity difference is amplified as participation grows.

Consider two agents with types

$$\omega' > \omega.$$

Suppose that the marginal value of coordination capacity is increasing in type,

$$B_{K\omega} > 0.$$

Then higher-productivity agents place greater value on additional coordination capacity.

The following sorting result is useful.

Proposition 4 (Positive sorting into productive coordination networks)**.** *Suppose*

$$B_{K\omega} > 0,$$

and suppose provider participation costs satisfy a single-crossing condition in which

$$\frac{\partial}{\partial \omega} [c_j(\omega) - c_k(\omega)]$$

is weakly monotone. Then equilibrium participation can be represented by cutoffs in ω. In particular, when $a_j > a_k$, sufficiently strong single crossing implies that the set of types preferring j to k is an upper interval.

Proof. The payoff difference between providers j and k is

$$u_j - u_k = B(\omega, Aa_j n_j^\gamma) - B(\omega, Aa_k n_k^\gamma) - [c_j(\omega) - c_k(\omega)].$$

Because $a_j > a_k$ and $B_{K\omega} > 0$, the benefit from the productivity difference is increasing in ω. The single-crossing condition therefore implies that the payoff difference crosses zero at most once. Hence the set of types preferring j is an upper interval. □

The proposition does not require increasing returns.

Increasing returns become important when determining the stability and concentration of provider scale.

7.5 Increasing Returns and Concentration

Consider the symmetric case in which providers have identical participation costs and productivity,

$$a_j = a, \qquad c_j(\omega) = c(\omega).$$

Suppose for simplicity that a representative participant receives

$$B(K) = bK$$

from coordination capacity. Then the benefit from provider j is

$$baAn_j^\gamma.$$

The marginal effect of participation on the capacity available to every existing participant is

$$\frac{\partial K_j}{\partial n_j} = Aa\gamma n_j^{\gamma-1}.$$

The sign of the second derivative is

$$\frac{\partial^2 K_j}{\partial n_j^2} = Aa\gamma(\gamma - 1)n_j^{\gamma-2}.$$

Hence:

$$\gamma < 1 \quad \Rightarrow \quad K_j \text{ is concave in } n_j,$$

while

$$\gamma > 1 \quad \Rightarrow \quad K_j \text{ is convex in } n_j.$$

The latter creates a force toward concentration.

In particular, when

$$\gamma > 1,$$

a provider that acquires an initially larger participant base enjoys a coordination-capacity advantage that increases with its scale.

This generates a coordination analogue of increasing returns in production.

7.6 A Network-Concentration Result

Let

$$r_j = \frac{n_j}{\sum_k n_k}$$

denote the share of network-coordinated agents associated with provider j, and define the Herfindahl concentration index

$$H_C = \sum_{j=1}^{J} r_j^2.$$

The following result characterizes the force generated by increasing returns.

Proposition 5 (Increasing returns generate a concentration force)**.** *Suppose that coordination capacity is*

$$K_j = Aa_j n_j^\gamma$$

with $\gamma > 1$, and suppose provider payoffs are increasing in K_j. Consider two providers j and k with equal productivity and positive participation. A perturbation that transfers a sufficiently small mass of participants from the smaller provider to the larger provider raises the coordination-capacity advantage of the larger provider.

Consequently, an interior symmetric allocation is unstable whenever the coordination-capacity feedback dominates the direct participation cost difference.

Proof. Let $n_j > n_k$. A transfer $\varepsilon > 0$ from k to j changes the capacity difference by

$$\Delta K = Aa\left[(n_j + \varepsilon)^\gamma - (n_k - \varepsilon)^\gamma - n_j^\gamma + n_k^\gamma\right].$$

Differentiating with respect to ε at zero gives

$$\frac{d\Delta K}{d\varepsilon} = Aa\gamma\left(n_j^{\gamma-1} + n_k^{\gamma-1}\right) > 0.$$

Because $\gamma > 1$, the marginal capacity contribution of the larger provider is increasing in its scale. Thus the transfer increases the relative capacity advantage of the larger provider. If the resulting payoff increase exceeds the corresponding participation-cost differential, the perturbation is profitable. Hence the symmetric interior allocation is unstable under the stated dominance condition. $\square$

The proposition establishes a force toward concentration but not a universal monopoly result.

Entry costs, heterogeneous productivity, congestion, multi-homing, and provider-specific participation costs can prevent complete concentration.

The relevant prediction is therefore

$$\boxed{\gamma \uparrow \quad \Longrightarrow \quad \text{stronger endogenous concentration forces,}}$$

not

$$\gamma \uparrow \quad \Longrightarrow \quad H_C = 1.$$

7.7 The Endogenous Organizational Boundary

The network-formation equilibrium determines the endogenous network cost

$$C_N^{**}(z;\theta) = C_N\left(z, G^*(z;\theta);\theta\right).$$

The organizational problem is therefore

$$C^{**}(z;\theta) = \min\left\{C_M^*(z), C_N^{**}(z;\theta), C_H^*(z)\right\}.$$

Define

$$D_{MN}^{**}(z;\theta) = C_N^{**}(z;\theta) - C_M^*(z)$$

and

$$D_{NH}^{**}(z;\theta) = C_H^*(z) - C_N^{**}(z;\theta).$$

The organizational boundaries satisfy

$$D_{MN}^{**}(z_{MN}^{**};\theta) = 0$$

and

$$D_{NH}^{**}(z_{NH}^{**};\theta) = 0.$$

We can now state the central boundary result.

[General organizational-boundary theorem] Suppose that:

1. $C_M^*(z)$ and $C_H^*(z)$ are continuous and differentiable;

2. $C_N^{**}(z;\theta)$ is continuous and differentiable in a neighborhood of an interior pair

$$z_{MN}^{**} < z_{NH}^{**};$$

3. at the market–network boundary,

$$\frac{\partial}{\partial z}\left(C_N^{**} - C_M^*\right) > 0;$$

4. at the network–hierarchy boundary,

$$\frac{\partial}{\partial z}\left(C_H^* - C_N^{**}\right) < 0.$$

Then there exists a neighborhood of the equilibrium in which the organizational allocation is locally ordered as

$$M \quad \longrightarrow \quad N \quad \longrightarrow \quad H,$$

with the boundaries satisfying

$$\frac{dz_{MN}^{**}}{d\theta_k} = -\frac{\partial_{\theta_k}(C_N^{**} - C_M^*)}{\partial_z(C_N^{**} - C_M^*)}$$

and

$$\frac{dz_{NH}^{**}}{d\theta_k} = -\frac{\partial_{\theta_k}(C_H^* - C_N^{**})}{\partial_z(C_H^* - C_N^{**})}.$$

Thus the organizational boundaries are endogenous functions of both the coordination technology and the equilibrium network.

Proof. The regularity conditions imply that the two boundary equations satisfy the conditions of the implicit function theorem. Hence each boundary is locally a differentiable function of θ.

The assumed signs of the derivatives with respect to z imply that the market–network and network–hierarchy crossings are locally unique. Since

$$z_{MN}^{**} < z_{NH}^{**},$$

there exists an interval

$$z \in (z_{MN}^{**}, z_{NH}^{**})$$

in which

$$C_N^{**}(z;\theta) < \min\{C_M^*(z), C_H^*(z)\}.$$

Thus network coordination is locally optimal on that interval, yielding the ordering

$$M \to N \to H.$$

The comparative-static expressions follow directly from the implicit function theorem. □

The theorem is deliberately a local result. It does not impose global single-crossing conditions that would rule out multiple organizational transitions.

That restriction is important because heterogeneous coordination providers, network congestion, and increasing returns can generate non-monotone organizational costs.

7.8 Ownership and Coordination Boundaries

The central conceptual distinction of the theory can now be stated formally.

Let

$$\Pi_F$$

denote the partition of productive agents into ownership units, and let

$$G^*$$

denote the equilibrium coordination network.

Define ownership concentration as

$$H_F = \sum_{m=1}^{M} s_m^2,$$

where s_m is the share of productive activity belonging to ownership unit m.

Define coordination concentration as

$$H_C = \sum_{j=1}^{J} r_j^2,$$

where r_j is the share of network-coordinated activity associated with provider j.

There is no general identity between these quantities:

$$H_F \neq H_C.$$

More importantly, the model permits them to move in opposite directions.

To characterize this formally, consider a technological parameter θ_k. Then

$$\frac{dH_F}{d\theta_k}$$

captures the effect of technological change on ownership concentration, while

$$\frac{dH_C}{d\theta_k}$$

captures its effect on coordination concentration.

7.9 The Coordination–Ownership Separation Result

Suppose that an increase in portable coordination technology reduces the coordination advantage of common ownership.

Formally, suppose that for a relevant set of activities,

$$\frac{\partial}{\partial P}\left[C_H^*(z) - C_N^{**}(z; P, A, \gamma)\right] > 0.$$

Then increasing P expands the set of activities for which network coordination dominates hierarchical integration.

At the ownership margin, suppose that the cost of maintaining a separate ownership unit is lower when coordination can be supplied externally. Let the net ownership-integration cost be

$$I(z, P),$$

with

$$I_P < 0.$$

Then an increase in P weakly reduces the incentive to combine productive agents under common ownership.

The following theorem formalizes the resulting separation.

[Coordination–ownership separation] Suppose that, over an interval of coordination requirements, the following conditions hold:

1. an increase in portability P weakly lowers equilibrium network coordination cost;

2. the network–hierarchy boundary is regular and shifts toward higher coordination requirements as P increases;

3. external network coordination reduces the relative cost of independent ownership, so that

$$I_P < 0;$$

4. ownership concentration is strictly decreasing in the prevalence of independent ownership units.

Then, over any interval in which the relevant boundaries remain interior,

$$\frac{dH_F}{dP} < 0$$

whenever portability induces a positive measure of activities to separate from common ownership, while

$$\frac{dH_C}{dP}$$

is positive whenever the induced expansion of the network generates a sufficient increase in provider participation concentration.

Hence there exist parameter regions in which

$$\boxed{\frac{dH_F}{dP} < 0 \qquad \text{and} \qquad \frac{dH_C}{dP} > 0.}$$

In such regions, technological progress simultaneously decentralizes productive ownership and concentrates coordination provision.

Proof. By condition 2 and Theorem 7.7, an increase in P expands the set of activities for which network coordination dominates hierarchy.

Condition 3 implies that external coordination reduces the relative cost of maintaining independent ownership. Therefore, whenever the boundary crosses a positive-measure set of activities, the mass of productive activity held under separate ownership increases.

By condition 4, this strictly reduces ownership concentration:

$$\frac{dH_F}{dP} < 0.$$

The effect on coordination concentration is not mechanically signed. However, when the expansion of network participation is accompanied by increasing provider-level returns or sufficiently strong sorting toward high-productivity providers, Proposition 5 implies that the dispersion of provider shares can decrease.

Thus, for parameter values in which the concentration force dominates the equalizing effect of network expansion,

$$\frac{dH_C}{dP} > 0.$$

Both inequalities can therefore hold simultaneously. $\square$

The importance of the theorem is not that the opposite-signed derivatives must always occur. They do not.

The theorem instead establishes that the two concentration measures are structurally distinct and identifies a nonempty parameter region in which they move in opposite directions.

This is the central organizational implication of the model.

7.10 General Coordination-Scale Decoupling

The previous theorem concerns concentration. The same logic applies to organizational scale.

Let

$$S_F$$

denote average ownership-unit scale and

$$S_C$$

denote the scale of the coordination system accessible to a productive agent.

The network architecture permits

$$S_F \neq S_C.$$

Under the conditions of Theorem 7.9, a rise in portability can reduce the need for common ownership while increasing the number of agents coordinated through the same network.

This yields the following corollary.

[Decoupling of ownership and coordination scale] Under the conditions of Theorem 7.9, suppose additionally that network participation expands with P. Then there exists a parameter region in which

$$\frac{dS_F}{dP} < 0 \qquad \text{and} \qquad \frac{dS_C}{dP} > 0.$$

Thus technological progress can simultaneously reduce the scale of the average ownership unit and increase the scale of the coordination system.

Proof. The first inequality follows from the reduction in the need for common ownership implied by condition 3 of Theorem 7.9. The second follows from the expansion of network participation and the definition of S_C as the scale of the coordination system. □

7.11 A General Concentration Decomposition

The preceding result can be sharpened by decomposing changes in coordination concentration.

Recall

$$H_C = \sum_j r_j^2.$$

Differentiating gives

$$dH_C = 2\sum_j r_j\, dr_j.$$

The provider share can be decomposed into an entry component, a reallocation component, and a scale component. Writing

$$dr_j = dr_j^E + dr_j^R + dr_j^S,$$

we obtain

$$dH_C = 2\sum_j r_j dr_j^E + 2\sum_j r_j dr_j^R + 2\sum_j r_j dr_j^S.$$

The three terms have distinct interpretations.

The entry component captures changes in the number of active coordination providers.

The reallocation component captures movement of participants across existing providers.

The scale component captures changes in the relative scale of providers.

Technological progress can therefore affect concentration through several channels that need not have the same sign.

In particular, portability may increase network entry and thereby reduce concentration while increasing returns simultaneously induce sorting toward large providers and increase concentration.

The sign of

$$\frac{dH_C}{dP}$$

is therefore an equilibrium object rather than a primitive prediction.

This is why the concentration theorem above is stated as a conditional result.

7.12 A Limiting Case: Recovery of the Canonical Firm

The framework collapses to the canonical theory when the coordination network cannot economically substitute for ownership.

Suppose

$$P \to 0$$

and

$$A \to 0.$$

Then network coordination capacity vanishes:

$$K_j \to 0,$$

and therefore

$$C_N^{**}(z;\theta) \to \infty$$

for positive coordination requirements.

The organizational problem becomes

$$C^{**}(z) = \min \left\{ C_M^*(z), C_H^*(z) \right\}.$$

The canonical market–hierarchy theory is therefore recovered as a limiting case.

The new theory can consequently be represented schematically as

$$\boxed{\text{Canonical firm theory} = \text{zero-effective-network limit of the general model.}}$$

This nesting property is important for interpretation. The model does not discard the canonical theory; it identifies the conditions under which its binary organizational boundary ceases to exhaust the feasible set.

7.13 What the Network-Formation Result Establishes

The endogenous network analysis establishes four propositions about the organization of production.

First, coordination networks are not merely exogenous technological environments. Their scale and structure are equilibrium outcomes of participation decisions.

Second, increasing returns in coordination provision create a force toward concentration, but do not by themselves imply monopoly.

Third, the organizational boundary is endogenous to the equilibrium network:

$$z_{MN}^{**} = z_{MN}^{**}(\theta), \qquad z_{NH}^{**} = z_{NH}^{**}(\theta).$$

Fourth, ownership concentration and coordination concentration are distinct equilibrium objects:

$$\boxed{H_F \neq H_C.}$$

The resulting possibility,

$$\boxed{H_F \downarrow \qquad \text{while} \qquad H_C \uparrow,}$$

is the sharpest departure from the canonical theory.

It implies that a world with increasingly decentralized productive ownership can nevertheless become increasingly dependent on a small number of coordination infrastructures.

This possibility is especially relevant when technological progress raises the portability, productivity, or scale elasticity of coordination.

The theory therefore predicts that technological change may alter not simply the size of firms, but the relationship between ownership, coordination, and economic concentration itself.

The empirical implications of these results are developed in the next section.

8 Empirical Implications and Measurement

The preceding sections developed a theory in which the technology of coordination affects organizational boundaries through an endogenous network of coordination providers. The principal theoretical implication is that productive ownership and coordination provision are distinct equilibrium objects. Consequently, empirical work motivated by the model should distinguish changes in the concentration of productive ownership from changes in the concentration of the infrastructures through which production is coordinated.

This section develops the empirical implications that follow directly from the model. It distinguishes theorem-backed predictions from broader hypotheses that would require extensions of the baseline framework.

8.1 Structural Objects

The model is governed by the structural vector

$$\theta = (P, A, \gamma).$$

These parameters have distinct economic interpretations.

The parameter

$$P$$

measures the portability of coordination capital across ownership boundaries. It affects the cost

$$\tau(P)$$

of sustaining persistent coordination relationships without common ownership.

The parameter

$$A$$

measures the productivity and generality of coordination capacity once a coordination infrastructure is deployed.

The parameter

$$\gamma$$

measures the elasticity of coordination capacity with respect to provider scale:

$$\frac{\partial \log K_j}{\partial \log n_j} = \gamma.$$

These objects are structural primitives rather than directly observed variables. Empirical analysis therefore requires either proxies or structural estimation.

The principal equilibrium outcomes are

$$G^*, \qquad \sigma^*, \qquad H_F, \qquad H_C, \qquad \Delta_B.$$

The theory is most naturally tested through the joint behavior of these objects rather than through any single outcome.

8.2 Prediction 1: Expansion of Network Organization

Proposition 1 and Theorem 7.7 imply that improvements in portable or general-purpose coordination technology expand the set of activities for which persistent network coordination is optimal, under the regularity conditions stated there.

Let

$$S_N$$

denote the share of productive activity organized through persistent networks.

Then the model predicts

$$\boxed{\frac{\partial S_N}{\partial P} > 0, \qquad \frac{\partial S_N}{\partial A} > 0,}$$

whenever the organizational boundaries remain interior and the relevant technological improvements lower equilibrium network coordination cost.

The empirical counterpart is an increase in productive activity coordinated through persistent cross-boundary infrastructures rather than exclusively through spot markets or vertically integrated firms.

Examples of empirical measures include the share of transactions mediated by long-lived digital coordination systems, the prevalence of independent contractors operating through persistent networks, or the fraction of productive relationships maintained outside common ownership.

8.3 Prediction 2: Decoupling of Ownership and Coordination Boundaries

The central new object introduced by the theory is the organizational-boundary gap

$$\Delta_B.$$

Recall that

$$\Delta_B = \sum_i \omega_i \mu \left(B_C(i; G^*) \setminus B_F(i; \Pi_F) \right).$$

The model predicts that improvements in the technology of distributed coordination can increase this gap:

$$\boxed{\frac{\partial \Delta_B}{\partial P} \geq 0, \qquad \frac{\partial \Delta_B}{\partial A} \geq 0,}$$

under the conditions that expand network organization.

This prediction is distinct from a prediction about firm size.

A productive agent may remain outside a common ownership unit while becoming more deeply embedded in a persistent coordination system.

Empirically, this suggests measuring the difference between ownership relationships and economically relevant coordination relationships. Examples include supplier networks, software ecosystems, professional communities, creator networks, or other infrastructures in which participants remain legally independent while coordinating repeatedly.

8.4 Prediction 3: Divergence of Ownership and Coordination Concentration

Theorem 7.9 establishes that there are parameter regions in which technological progress simultaneously reduces ownership concentration and increases coordination concentration.

The signature prediction of the model is therefore

$$\boxed{\frac{dH_F}{dP} < 0 \qquad \text{and} \qquad \frac{dH_C}{dP} > 0,}$$

for economies satisfying the conditions of the theorem.

This is a joint prediction.

The theory does not predict that ownership concentration must always fall or that coordination concentration must always rise. Instead, it predicts that the two measures need not move together and identifies conditions under which they move in opposite directions.

Empirical analysis should therefore examine the covariance between

$$H_F$$

and

$$H_C$$

rather than relying on ownership concentration alone.

The relevant coordination providers may include platforms, digital infrastructures, enterprise software systems, cloud services, or other organizations that supply persistent coordination capacity.

8.5 Prediction 4: Increasing Returns and Coordination Concentration

Proposition 5 establishes that increasing returns in coordination provision create a force toward concentration.

The prediction is not

$$\gamma > 1 \Rightarrow H_C = 1.$$

Entry, congestion, multi-homing, and heterogeneous provider costs can offset the concentration force.

The theorem-backed prediction is instead

$$\boxed{\text{larger } \gamma \quad \Longrightarrow \quad \text{stronger endogenous concentration forces in coordination provision.}}$$

Empirically, sectors in which coordination capacity scales more than proportionally with participant base should exhibit stronger tendencies toward concentration among coordination providers, all else equal.

This prediction concerns the concentration of coordination infrastructure, not necessarily the concentration of productive firms.

8.6 Prediction 5: Heterogeneous Effects Across Activities

The organizational boundaries depend on the coordination requirement

$$z.$$

Consequently, the effects of improvements in P or A should be heterogeneous across activities.

Activities near the market–network boundary are the most likely to move from market coordination into persistent networks.

Activities near the network–hierarchy boundary are the most likely to move from hierarchy into network coordination.

Activities far from either boundary should be relatively unaffected.

The model therefore predicts that technological improvements in coordination should have the largest organizational effects for activities with intermediate coordination requirements rather than uniformly across the economy.

This heterogeneity is a direct implication of the boundary formulation and provides a natural basis for empirical designs exploiting variation in coordination intensity across tasks or industries.

8.7 Prediction 6: Threshold Effects

The baseline model allows increasing returns in coordination provision through

$$K_j = Aa_j n_j^{\gamma}.$$

When these increasing returns interact with participation decisions, network-formation equilibria can exhibit multiple locally stable allocations.

The theory therefore permits threshold effects:

$$\boxed{\text{small technological changes may produce discrete organizational changes}}$$

when the economy crosses a network-formation threshold.

This prediction is conditional rather than universal. It requires that the network-formation feedback be sufficiently strong to generate multiple local equilibria or instability of interior symmetric allocations.

Empirically, this suggests looking for nonlinear responses in organizational form around technological or institutional changes that materially reduce the cost of distributed coordination.

8.8 Measurement of Ownership Concentration

Ownership concentration is the more familiar empirical object.

Let

$$s_m$$

denote the share of productive activity or value added controlled by ownership unit m.

Then

$$H_F = \sum_m s_m^2.$$

Depending on the application, s_m can be measured using employment, revenue, value added, assets, or output.

The theory does not require a particular ownership measure, provided the same measure is used consistently over time.

8.9 Measurement of Coordination Concentration

Coordination concentration is less conventional and is therefore one of the paper's empirical contributions.

Let

$$r_j$$

denote the share of economically relevant coordination capacity supplied by provider j.

Then

$$H_C = \sum_j r_j^2.$$

The appropriate empirical measure depends on the application.

Possible proxies include:

- the share of transactions mediated by a coordination provider;

- the share of users relying on a common digital infrastructure;
- the share of productive relationships organized through a common platform;
- the share of enterprises using a common enterprise coordination system;
- the share of network interactions passing through a provider.

These measures need not coincide with ownership shares.

Indeed, the theory predicts that they may diverge systematically.

8.10 Measurement of the Boundary Gap

The organizational-boundary gap

$$\Delta_B$$

requires observing both ownership relationships and coordination relationships.

Let

$$\mathcal{O}$$

denote the graph of ownership relationships and

$$\mathcal{C}$$

the graph of economically relevant coordination relationships.

A sample analogue of the boundary gap can be constructed by measuring the fraction of coordination links that connect agents belonging to different ownership units.

For example,

$$\widehat{\Delta}_B = \frac{|\mathcal{C} \setminus \mathcal{O}|}{|\mathcal{C}|}.$$

This measure equals zero when economically relevant coordination is entirely contained within ownership boundaries and rises as coordination increasingly crosses those boundaries.

The boundary-gap measure is potentially useful because it directly captures the theoretical distinction between ownership and coordination.

8.11 Structural Estimation

Because

$$P, \quad A, \quad \gamma$$

are latent primitives, reduced-form regressions alone cannot identify them without additional assumptions.

A structural approach would proceed by specifying the network-formation model

$$G^*(z; \theta)$$

and estimating the parameter vector

$$\theta = (P, A, \gamma)$$

from observed participation decisions, provider scales, and organizational boundaries.

The relevant moments could include:

network participation rates,

provider size distributions,

ownership-unit size distributions,

coordination concentration H_C,

ownership concentration H_F,

and boundary-gap measures Δ_B.

The model is particularly well suited to environments in which both organizational structure and network participation can be observed.

8.12 Reduced-Form Tests

When structural estimation is infeasible, the theory still suggests a reduced- form empirical strategy.

Suppose an observable proxy

$$\widetilde{P}_{it}$$

captures changes in portable coordination technology.

One may estimate

$$Y_{it} = \alpha_i + \lambda_t + \beta \widetilde{P}_{it} + X_{it}' \rho + \varepsilon_{it},$$

where Y_{it} is one of the equilibrium outcomes predicted by the model.

The most direct outcomes are

$$S_N, \qquad \Delta_B, \qquad H_F, \qquad H_C.$$

The theory predicts a coherent pattern across these outcomes rather than a single coefficient.

In particular, a change that genuinely increases portable coordination should, under the conditions of the model, be associated with

$$S_N \uparrow,$$

$$\Delta_B \uparrow,$$

and, in the parameter region described by Theorem 7.9,

$$H_F \downarrow, \qquad H_C \uparrow .$$

Observing this joint pattern would provide considerably stronger evidence for the theory than observing any one outcome in isolation.

8.13 Distinguishing the Theory from Conventional Productivity Shocks

A conventional productivity shock raises the productivity of production.

The theory developed here predicts a different empirical pattern because the shock operates through coordination.

A pure production-productivity improvement need not increase

$$\Delta_B$$

or expand the network region.

By contrast, an improvement in P or A should affect organizational boundaries even if the underlying production technology remains unchanged.

This distinction suggests a way to separate the theory empirically from standard productivity explanations.

Technological changes that primarily affect communication, information portability, or coordination capacity should have stronger effects on organizational structure than technologies that merely increase the efficiency of a particular production process.

8.14 Summary of Theorem-Backed Predictions

The principal empirical implications of the model are summarized below.

Prediction	**Theoretical implication**
Network expansion	Higher P or A expands the set of activities for which persistent network coordination is optimal.
Boundary decoupling	The coordination boundary can expand beyond the ownership boundary, increasing Δ_B.
Concentration divergence	There exist parameter regions in which ownership concentration falls while coordination concentration rises.
Increasing returns	Higher γ strengthens endogenous concentration forces among coordination providers.
Heterogeneous effects	Organizational changes are largest for activities near the market–network or network–hierarchy boundaries.
Threshold effects	Strong network feedback can generate discontinuous organizational changes around network-formation thresholds.

Table 1: Theorem-backed empirical implications of the model.

These predictions are deliberately narrower than a list of general consequences of social media or artificial intelligence.

They arise from the structural distinction between productive ownership and coordination provision.

The central empirical question posed by the theory is therefore not simply whether new technologies make firms larger or smaller. It is whether they alter the relationship between the ownership of productive activity and the infrastructures through which that activity is coordinated.

The remainder of the paper discusses the broader economic implications of this distinction and the limits of the baseline framework.

9 Economic Implications and Limits

The preceding analysis establishes a general organizational framework in which productive ownership and economic coordination are distinct margins. The distinction is consequential because technological change can affect these margins differently. This section draws out the broader implications of the framework and clarifies the limits of the baseline model.

The central implication is not that markets, networks, or hierarchies are inherently superior organizational forms. Rather, the optimal boundary of the firm depends on the relative technologies through which economic activity can be coordinated. When coordination becomes portable, productive, and scalable, the boundary of ownership need not remain the natural boundary of coordination.

9.1 The Firm as an Ownership Institution

The canonical theory of the firm treats the firm primarily as an alternative to market exchange for organizing transactions.

The present framework retains this interpretation but separates two questions that are often jointly embedded in the canonical formulation:

1. Who owns the productive assets and retains residual control rights?

2. Through what infrastructure are economically interdependent agents coordinated?

In the baseline model, these questions need not have the same answer.

Let

$$\Pi_F$$

denote the ownership partition and

$$G^*$$

the equilibrium coordination network.

Then an economic system can exhibit

$$\Pi_F \neq \Pi(G^*).$$

This possibility does not eliminate the firm as an economic institution.

Ownership remains important because it determines residual control, investment incentives, and the allocation of decision rights. What changes is the relationship between ownership and coordination.

The firm is therefore no longer necessarily the smallest natural unit within which all economically important coordination occurs.

9.2 Coordination as a General-Purpose Input

A central implication of the framework is that coordination itself can be treated as a productive input.

The coordination technology is

$$K_j = Aa_j n_j^{\gamma}.$$

This formulation is deliberately analogous to a production technology. It implies that coordination capacity can be produced, accumulated, deployed, and supplied to multiple productive agents.

The economic significance of this observation is that a coordination provider can specialize in producing coordination capacity without owning all of the productive assets that use it.

This creates a division of labor between production

and

coordination provision.

The distinction is conceptually broader than the existence of platforms. Platforms are one institutional form through which coordination capacity may be supplied.

The deeper implication is that the economy can support specialization not only in the production of goods and services but also in the production of the organizational infrastructure through which other producers coordinate.

9.3 A New Source of Economies of Scope

The general-purpose nature of coordination capacity creates a potential source of economies of scope.

Suppose a coordination provider can support multiple productive activities using the same underlying infrastructure.

Let

$$K_j = A \left(\sum_{\ell=1}^{L} n_{j\ell} \right)^{\gamma},$$

where $n_{j\ell}$ denotes participation associated with productive activity ℓ.

Then the same coordination infrastructure can potentially be reused across activities.

This creates a distinction between activity-specific economies of scale and general-purpose coordination economies.

A conventional production technology may exhibit

$$F_\ell(x_\ell)$$

for each activity ℓ, while a common coordination infrastructure generates

$$K_j = K_j(n_{j1}, \ldots, n_{jL}).$$

If coordination capacity is sufficiently general-purpose, the marginal cost of coordinating an additional activity can fall as the total number of activities served by the infrastructure increases.

This provides one reason why the organizational consequences of general-purpose technologies can extend beyond the industries in which those technologies are directly produced.

9.4 The New Meaning of Firm Size

The theory implies that firm size becomes an incomplete statistic for organizational scale.

Let

$$S_F$$

denote the scale of productive ownership and

$$S_C$$

the scale of coordination.

The canonical interpretation tends to emphasize

$$S_F.$$

The present framework requires both:

$$(S_F, S_C).$$

It is possible to have

$$S_F \downarrow \qquad \text{and} \qquad S_C \uparrow .$$

An economy can therefore become more decentralized according to conventional measures of firm size while simultaneously becoming more integrated through large coordination infrastructures.

This distinction changes the interpretation of several familiar empirical phenomena.

An increase in the number of firms need not imply greater economic decentralization if those firms increasingly depend on a small number of coordination infrastructures.

Conversely, a large coordination provider need not imply a large vertically integrated firm if the productive agents using its infrastructure remain independently owned.

The appropriate empirical question is therefore not simply

How large are firms?

but

How large are the systems through which firms coordinate?

9.5 Concentration Without Vertical Integration

The distinction between H_F and H_C implies a new form of economic concentration.

Traditional measures of concentration largely concern ownership or control over productive assets.

The present framework introduces

$$H_C = \sum_j r_j^2$$

as concentration in coordination provision.

An economy can therefore experience

$$H_F \downarrow$$

while

$$H_C \uparrow .$$

This is neither ordinary vertical integration nor ordinary horizontal concentration.

Instead, productive ownership becomes more dispersed while coordination provision becomes more concentrated.

The distinction matters for economic analysis because concentration in coordination infrastructure may affect the economic environment even when traditional measures of firm concentration suggest increasing decentralization.

The model therefore suggests that the relevant unit for some questions of market power may be neither the conventional firm nor the conventional industry, but the provider of a critical coordination infrastructure.

9.6 The Economics of Portability

Portability is a particularly important primitive because it changes the relationship between productive agents and ownership institutions.

Recall that

$$\tau(P)$$

is the cost of sustaining coordination across ownership boundaries.

An increase in P lowers this cost:

$$\tau'(P) < 0.$$

When portability is low, productive relationships are relatively dependent on the institutions in which they were initially embedded.

When portability is high, productive agents can retain coordination capital while moving across ownership boundaries.

This has implications beyond digital platforms.

Portability can apply to:

- reputation;
- professional relationships;
- customer relationships;
- accumulated knowledge;
- software environments;
- organizational routines;
- communication networks;
- AI-assisted workflows.

The common economic feature is that valuable coordination capacity is no longer necessarily destroyed when an agent changes organizational affiliation.

This weakens one of the historical economic advantages of vertical integration: the ability of the organization to preserve coordination relationships by keeping the relevant agents together.

9.7 Artificial Intelligence and the Boundary of the Firm

Artificial intelligence is potentially important within the framework because it can alter coordination technology itself.

Suppose an AI system performs tasks such as

$$\{\text{matching, monitoring, planning, information aggregation, communication, forecasting}\}.$$

These functions can increase the productivity of coordination capacity and therefore raise A.

If the same AI infrastructure can serve increasingly large populations of productive agents, technological progress may also raise γ.

The organizational implications are therefore ambiguous in sign.

An increase in A can make distributed coordination more attractive:

$$A \uparrow \quad \Rightarrow \quad C_N^{**} \downarrow .$$

But an increase in γ can simultaneously strengthen concentration among coordination providers:

$$\gamma \uparrow \quad \Rightarrow \quad \text{stronger concentration forces.}$$

AI can consequently produce two apparently contradictory outcomes:

$$\boxed{\text{greater decentralization of productive ownership}}$$

and

$$\boxed{\text{greater concentration of coordination infrastructure.}}$$

The model therefore does not imply that AI will make firms uniformly larger or smaller.

Its organizational effect depends on which component of coordination technology it changes.

9.8 Social Media and the Portability of Coordination Capital

Social media provides a complementary mechanism.

Its most direct interpretation in the model is through portability.

Suppose social technologies make relationships, audiences, reputational information, and communication channels less dependent on a particular ownership structure. Then

$$P \uparrow .$$

The resulting reduction in

$$\tau(P)$$

makes persistent cross-boundary coordination cheaper.

The model therefore provides a theoretical interpretation of an empirical phenomenon in which individuals can remain economically productive while maintaining relationships with large populations of customers, collaborators, or audiences without being incorporated into a common ownership structure.

The key implication is again not that social media "eliminates firms."

Rather,

$$\boxed{\text{social technology can weaken the necessity of ownership for coordination.}}$$

This is precisely the margin that the canonical market–hierarchy framework does not treat as a separate organizational technology.

9.9 Implications for the Division of Labor

The theory also has implications for the division of labor.

In a conventional framework, specialization is often accompanied by organizational integration because specialized productive units require coordination.

The present framework implies that these two forces can be separated.

Suppose specialization increases the coordination requirement

$$z.$$

In a world with low coordination portability, the resulting increase in z may favor hierarchical integration.

In a world with sufficiently productive network coordination, the same increase in specialization can instead increase demand for external coordination capacity.

Thus the effect of specialization on firm boundaries depends on the state of coordination technology.

The model therefore predicts that improvements in general-purpose coordination technology can change not only the allocation of production across tasks but also the organizational form through which specialization is implemented.

9.10 Implications for Market Power

The framework also suggests a distinction between productive market power and coordination-infrastructure power.

A coordination provider may have substantial influence over economic activity because many productive agents depend on its infrastructure even when the provider owns relatively little productive capital itself.

This distinction can be represented by

$$H_C$$

being high while

$$H_F$$

is low.

The theory does not by itself establish that such concentration creates market power in the antitrust sense. That requires an additional model of pricing, exclusion, entry, or strategic behavior.

It does, however, identify a potentially important object of economic concentration that is invisible to ownership-based measures.

This provides a natural direction for extensions in which coordination providers choose prices, access rules, interoperability, or exclusion.

9.11 General-Purpose Coordination and the Future Firm

The deepest implication of the framework concerns the role of general-purpose coordination technology in determining organizational form.

Let the effective coordination technology be summarized by

$$\mathcal{K} = (P, A, \gamma).$$

The canonical theory implicitly treats the ability to coordinate across ownership boundaries as sufficiently limited that ownership remains a central mechanism for preserving productive coordination.

The present framework allows

$$\mathcal{K}$$

to become sufficiently powerful that this identification breaks down.

The resulting economy can contain three simultaneously distinct scales:

$$\boxed{\text{production scale,} \qquad \text{ownership scale,} \qquad \text{coordination scale.}}$$

There is no theoretical requirement that these scales coincide.

This suggests a broader conception of the firm.

The firm remains an ownership institution, but it becomes one component of a larger organizational system in which productive agents, ownership units, and coordination infrastructures can be separately specialized.

The economically relevant unit of organization may consequently become a network of firms rather than a single firm.

9.12 Limits of the Baseline Theory

The baseline framework abstracts from several mechanisms that may matter in applications.

First, coordination providers are not modeled as strategic price setters. The current theory therefore does not establish implications for markups or consumer welfare.

Second, the baseline model treats the productivity of coordination providers through a_j, while more complicated environments may involve endogenous investment in coordination infrastructure.

Third, the model abstracts from privacy, security, political constraints, regulation, and institutional barriers that may limit portability.

Fourth, the network-formation analysis focuses on participation and scale. Richer models may need to incorporate directed links, heterogeneous link costs, multi-homing, congestion, or endogenous provider entry.

Fifth, the model does not assume that artificial intelligence necessarily increases P, A, or γ. Those are empirical questions. AI is therefore an application of the framework rather than a primitive assumption required for its validity.

These limitations define natural directions for extensions rather than weakening the core organizational result.

9.13 What the Theory Changes

The central claim of the paper can now be stated compactly.

The canonical theory of the firm asks:

$$\boxed{\text{When should production be organized through markets rather than firms?}}$$

The present framework asks a more general question:

$$\boxed{\text{Who should own production, and who should provide the coordination infrastructure through which pro}}$$

When coordination is sufficiently portable, productive, and scalable, the answer to these questions need not be the same institution.

The resulting organizational architecture is characterized by

$$\boxed{\text{ownership } \perp \text{ coordination}}$$

in the sense that the two boundaries need not coincide.

This provides a general explanation for why technological progress can produce the seemingly paradoxical combination of

$$\text{smaller ownership units,} \qquad \text{larger coordination systems,} \qquad \text{and greater concentration in coordination p}$$

The theory therefore predicts not simply a new type of firm, but a change in the relationship between firms and the organizational infrastructure of the economy.

9.14 Conclusion

The framework developed in this paper extends the theory of the firm by treating coordination as a separately scalable economic technology.

Once coordination can be portable across ownership boundaries, productive in its own right, and subject to increasing returns, the traditional equivalence between ownership scale and coordination scale no longer follows.

The resulting economy can simultaneously exhibit decentralized productive ownership and concentrated coordination infrastructure.

Social media and artificial intelligence are important not because the theory is tied to either technology, but because they represent particularly powerful instances of technologies that may increase the portability, productivity, and scale elasticity of coordination.

The broader theoretical implication is therefore:

$$\boxed{\text{The boundary of the firm is endogenous to the technology of coordination.}}$$

The canonical market–hierarchy framework emerges when external coordination is sufficiently costly. The generalized framework becomes relevant when coordination itself can be produced and supplied across organizational boundaries.

The theory thus provides a foundation for analyzing an economy in which the organizational problem is no longer exhausted by the choice between markets and firms, but includes the endogenous emergence of scalable coordination networks between them.

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A Mathematical Foundations and Equilibrium Existence

This appendix provides the mathematical foundations for the organizational problem developed in Sections 4–7. It formalizes the feasible organizational architectures, states the regularity conditions required for the main results, establishes existence of the equilibrium organizational choice, and derives the local regularity of the organizational boundaries.

The appendix is deliberately explicit about the assumptions required for each result. In particular, the existence of an equilibrium network does not by itself imply uniqueness, stability, or global monotonicity. Those stronger properties require additional conditions and are treated separately in Appendix B.

A.1 A.1. Primitive Environment

Let the set of productive agents be

$$\mathcal{I} = [0, 1]$$

equipped with the Borel σ-algebra and Lebesgue measure μ.

Each productive agent $i \in \mathcal{I}$ has type

$$\omega_i \in \Omega,$$

where

$$\Omega \subset \mathbb{R}_+$$

is compact.

The type ω_i summarizes the productive and organizational characteristics of agent i that are relevant for the coordination problem. The baseline results do not require a particular interpretation of ω; it may represent productivity, coordination intensity, specialization, or a sufficient statistic for the agent's value from coordination.

The economy contains a finite set of potential coordination providers

$$\mathcal{J} = \{1, \ldots, J\}.$$

Provider j has coordination productivity

$$a_j > 0.$$

The structural coordination technology is

$$K_j = A a_j n_j^{\gamma},$$

where

$$A > 0, \qquad \gamma > 0,$$

and n_j is the measure of productive agents coordinated by provider j.

The parameter A captures the productivity of the underlying coordination technology, while γ captures the elasticity of coordination capacity with respect to provider scale.

The portability parameter satisfies

$$P \in \mathcal{P} \subseteq \mathbb{R}_+.$$

Cross-boundary coordination incurs a cost

$$\tau(P),$$

with

$$\tau'(P) \leq 0.$$

Strict inequality is imposed only where comparative-static results require strict improvements in portability.

A.2 A.2. Organizational Architectures

The organizational choice set is

$$\mathcal{S} = \{M, N, H\},$$

where

$$M = \text{market coordination},$$

$$N = \text{network coordination},$$

and

$$H = \text{hierarchical coordination.}$$

For a coordination requirement $z \in \mathcal{Z} \subseteq \mathbb{R}_+$, define the minimized costs under the three architectures as

$$C_M^*(z), \qquad C_H^*(z), \qquad C_N^*(z;\theta),$$

where

$$\theta = (P, A, \gamma).$$

The network cost depends additionally on an equilibrium network G^*. Accordingly, after network formation has been solved, define

$$C_N^{**}(z;\theta) = C_N(z, G^*(z;\theta); \theta).$$

The full organizational problem is therefore

$$C^{**}(z;\theta) = \min_{s \in \mathcal{S}} C_s^{**}(z;\theta),$$

where

$$C_M^{**} = C_M^*, \qquad C_H^{**} = C_H^*, \qquad C_N^{**} = C_N(z, G^*; \theta).$$

The equilibrium organizational correspondence is

$$\mathcal{S}^*(z;\theta) = \arg\min_{s \in \mathcal{S}} C_s^{**}(z;\theta).$$

When the minimizer is unique, write

$$s^*(z;\theta)$$

for the equilibrium organizational architecture.

A.3 A.3. Regularity Conditions

The following assumptions provide the baseline regularity conditions used throughout the appendices.

Assumption 2 (Compact type and coordination spaces)**.** *The type space Ω and the relevant coordination-requirement space $\mathcal{Z}$ are compact subsets of $\mathbb{R}_+$.*

Assumption 3 (Continuity of coordination benefits)**.** *The coordination-benefit function*

$$B : \Omega \times \mathbb{R}_+ \to \mathbb{R}$$

is continuous and continuously differentiable in its arguments, with

$$B_K(\omega, K) > 0.$$

Where sorting results are invoked, we additionally impose

$$B_{K\omega}(\omega, K) \geq 0.$$

Assumption 4 (Participation costs)**.** *For each $j \in \mathcal{J}$, the participation cost*

$$c_j : \Omega \to \mathbb{R}$$

is continuous. Where monotone sorting is invoked, the relevant cost differences satisfy the single-crossing condition stated in the corresponding proposition.

Assumption 5 (Market and hierarchical costs)**.** *The minimized market and hierarchical coordination costs,*

$$C_M^*(z) \quad and \quad C_H^*(z),$$

are finite and continuously differentiable on the interior of $\mathcal{Z}$.

Assumption 6 (Network-cost regularity)**.** *For every feasible network G, the network coordination cost*

$$C_N(z, G; \theta)$$

is finite and continuous in (z, G, θ) and continuously differentiable in (z, θ) wherever the relevant equilibrium network is locally unique.

Assumption 7 (Portability)**.** *The cross-boundary coordination cost $\tau(P)$ is continuous and weakly decreasing in P:*

$$\tau'(P) \leq 0.$$

Where strict comparative statics with respect to portability are invoked,

$$\tau'(P) < 0.$$

A.4 A.4. Feasible Network Allocations

A network allocation is a measurable function

$$g : \mathcal{I} \to \mathcal{J} \cup \{0\}.$$

The interpretation is

$$g(i) = \begin{cases} j, & \text{if agent } i \text{ participates in provider } j, \\ 0, & \text{if agent } i \text{ uses decentralized market coordination.} \end{cases}$$

The participation measure of provider j is

$$n_j(g) = \mu\left(\{i \in \mathcal{I} : g(i) = j\}\right).$$

The vector of provider scales is

$$n(g) = (n_1(g), \ldots, n_J(g)).$$

Because the total mass of productive agents is normalized to one,

$$0 \leq n_j(g) \leq 1$$

and

$$\sum_{j=1}^{J} n_j(g) \leq 1.$$

Define the feasible scale simplex

$$\mathcal{N} = \left\{ n \in [0,1]^J : \sum_{j=1}^{J} n_j \leq 1 \right\}.$$

The coordination capacity vector is then

$$K(n;\theta) = \left(Aa_1 n_1^{\gamma}, \ldots, Aa_J n_J^{\gamma}\right).$$

Since $\mathcal{N}$ is compact and the capacity function is continuous for $\gamma > 0$, the induced capacity correspondence is well-defined.

A.5 A.5. Individual Best Responses

Given a scale vector $n \in \mathcal{N}$, the payoff from choosing provider j is

$$u_j(\omega; n, \theta) = B\left(\omega, A a_j n_j^{\gamma}\right) - c_j(\omega).$$

The market payoff is

$$u_0(\omega; n, \theta) = u_M(\omega).$$

The individual best-response correspondence is

$$BR(\omega; n, \theta) = \arg\max_{j \in \mathcal{J} \cup \{0\}} u_j(\omega; n, \theta).$$

Because $\mathcal{J} \cup \{0\}$ is finite and each payoff is continuous in the relevant arguments, the best-response correspondence is nonempty.

The equilibrium scale vector must satisfy the consistency condition

$$n_j = \mu\left(\{i : g(i) \in BR(\omega_i; n, \theta) \text{ and } g(i) = j\}\right).$$

Equivalently, define the aggregate best-response correspondence

$$\Phi(n; \theta) = \left\{\left(\mu(BR_j(n, \theta))\right)_{j=1}^{J} : \text{measurable tie-breaking}\right\},$$

where $BR_j(n, \theta)$ denotes the set of types for which j is optimal.

A network-formation equilibrium is a fixed point

$$n^* \in \Phi(n^*; \theta).$$

A.6 A.6. Existence of Network-Formation Equilibrium

The following proposition formalizes the existence claim used in Section 7.2.

Proposition 6 (Existence of network-formation equilibrium)**.** *Under Assumptions 2–7, the aggregate best-response correspondence admits a fixed point whenever its induced tie-breaking correspondence is measurable and upper hemicontinuous. Consequently, a network-formation equilibrium exists.*

Proof. The feasible scale set $\mathcal{N}$ is nonempty, compact, and convex.

For each provider j, the payoff function

$$u_j(\omega; n, \theta) = B(\omega, A a_j n_j^{\gamma}) - c_j(\omega)$$

is continuous in n by continuity of B and the continuity of n_j^{γ} on $[0, 1]$ for $\gamma > 0$.

Because the action set $\mathcal{J} \cup \{0\}$ is finite, the individual best-response correspondence is nonempty and upper hemicontinuous.

Under the stated measurability condition, the induced aggregate best-response correspondence Φ is nonempty and upper hemicontinuous. Its values are compact and convex under measurable randomization or measurable tie-breaking.

Kakutani's fixed-point theorem therefore implies that there exists

$$n^* \in \mathcal{N}$$

such that

$$n^* \in \Phi(n^*; \theta).$$

This is a network-formation equilibrium. □

The proposition establishes existence but intentionally does not establish uniqueness.

In particular, when $\gamma > 1$, the feedback between participation and coordination capacity can generate multiple equilibria. This possibility is economically meaningful and is treated explicitly in Appendix B.

A.7 A.7. Equilibrium Organizational Choice

Given an equilibrium network $G^*(z;\theta)$, define

$$C_N^{**}(z;\theta) = C_N(z, G^*(z;\theta);\theta).$$

The organizational choice is then made from the finite set

$$\mathcal{S} = \{M, N, H\}.$$

This immediately yields existence of an optimal organizational architecture.

Proposition 7 (Existence of an optimal organizational architecture)**.** *Suppose that an equilibrium network exists for every relevant z and θ, and that the corresponding network cost $C_N^{**}(z;\theta)$ is finite. Then the organizational problem*

$$\min_{s\in\{M,N,H\}} C_s^{**}(z;\theta)$$

admits a solution for every (z,θ).

Proof. The organizational choice set contains only three elements. Each organizational architecture has a finite cost by assumption. Hence the minimum over the finite set exists. □

Notice that the proposition does not require uniqueness. If two architectures have identical costs, the equilibrium organizational correspondence contains both.

This distinction is relevant at organizational boundaries.

A.8 A.8. Regular Organizational Boundaries

Define the market–network cost difference

$$D_{MN}(z;\theta) = C_N^{**}(z;\theta) - C_M^*(z)$$

and the network–hierarchy cost difference

$$D_{NH}(z;\theta) = C_H^*(z) - C_N^{**}(z;\theta).$$

A market–network boundary $z_{MN}^*(\theta)$ satisfies

$$D_{MN}(z_{MN}^*(\theta);\theta) = 0.$$

Similarly, a network–hierarchy boundary $z_{NH}^*(\theta)$ satisfies

$$D_{NH}(z_{NH}^*(\theta);\theta) = 0.$$

We say that a boundary is *regular* if the corresponding derivative with respect to z is nonzero.

Thus,

$$\frac{\partial D_{MN}}{\partial z} \neq 0$$

at a regular market–network boundary, and

$$\frac{\partial D_{NH}}{\partial z} \neq 0$$

at a regular network–hierarchy boundary.

Regularity rules out tangential intersections.

A.9 A.9. Local Comparative Statics of the Boundaries

The main-text organizational-boundary theorem follows locally from the implicit function theorem.

Proposition 8 (Local differentiability of organizational boundaries)**.** *Suppose $z^*_{MN}(\theta_0)$ is a regular market–network boundary and D_{MN} is continuously differentiable in a neighborhood of $(z^*_{MN}(\theta_0), \theta_0)$. Then there exists a neighborhood of θ_0 and a unique continuously differentiable function $z^*_{MN}(\theta)$ satisfying*

$$D_{MN}(z^*_{MN}(\theta); \theta) = 0.$$

Moreover,

$$\frac{\partial z^*_{MN}}{\partial \theta_k} = -\frac{\partial D_{MN}/\partial \theta_k}{\partial D_{MN}/\partial z}.$$

An analogous result holds for $z^*_{NH}(\theta)$.

Proof. At the boundary,

$$D_{MN}(z^*_{MN}(\theta_0); \theta_0) = 0.$$

By regularity,

$$\frac{\partial D_{MN}}{\partial z} \neq 0.$$

The implicit function theorem therefore guarantees the existence of a locally unique continuously differentiable function $z^*_{MN}(\theta)$ satisfying the boundary equation.

Differentiating

$$D_{MN}(z^*_{MN}(\theta); \theta) = 0$$

with respect to θ_k gives

$$\frac{\partial D_{MN}}{\partial z}\frac{\partial z^*_{MN}}{\partial \theta_k}+\frac{\partial D_{MN}}{\partial \theta_k}=0.$$

Rearranging yields

$$\frac{\partial z^*_{MN}}{\partial \theta_k}=-\frac{\partial D_{MN}/\partial \theta_k}{\partial D_{MN}/\partial z}.$$

The network–hierarchy result follows identically. □

A.10 A.10. The Canonical Limit

The canonical market–hierarchy model is recovered when network coordination becomes prohibitively costly.

Suppose that

$$\tau(P)\to\infty$$

or, equivalently for the reduced-form organizational problem,

$$C_N^{**}(z;\theta)\to\infty$$

for all $z>0$.

Then

$$C^{**}(z;\theta)\to\min\left\{C_M^*(z),C_H^*(z)\right\}.$$

Thus the network architecture disappears from the choice set in the limit.

Proposition 9 (Canonical market–hierarchy limit)**.** *If*

$$C_N^{**}(z;\theta) \to \infty$$

uniformly on compact subsets of $\mathcal{Z} \setminus \{0\}$, then the equilibrium organizational correspondence converges to

$$\arg \min_{s \in \{M,H\}} C_s^*(z).$$

Hence the canonical market–hierarchy problem is nested as a limiting case of the generalized organizational model.

Proof. For any $z > 0$, if

$$C_N^{**}(z;\theta) \to \infty,$$

then there exists a sufficiently large value of the relevant parameter such that

$$C_N^{**}(z;\theta) > \max\{C_M^*(z), C_H^*(z)\}.$$

The network architecture therefore cannot attain the minimum. The remaining problem is

$$\min\{C_M^*(z), C_H^*(z)\}.$$

Uniform convergence preserves this result over compact subsets away from $z = 0$. □

A.11 A.11. Separation of Existence, Uniqueness, and Stability

Because these concepts are sometimes conflated, it is useful to state explicitly what the baseline analysis establishes.

1. **Existence:** Under the regularity conditions above, at least one network-formation equilibrium exists.

2. **Uniqueness:** Existence does not imply uniqueness. In particular, increasing returns in coordination provision can generate multiple equilibria.

3. **Stability:** An equilibrium can exist without being stable under a particular dynamic adjustment process.

4. **Comparative statics:** Differentiability of organizational boundaries requires regularity of the relevant equilibrium and the corresponding cost crossing.

The distinction is important for interpreting the main-text results.

The baseline theory therefore does not rely on a global uniqueness assumption. Where a theorem requires local uniqueness or stability, that condition is stated explicitly.

A.12 A.12. Summary of the Mathematical Structure

The mathematical structure of the model can be summarized as the sequence

$$\theta = (P, A, \gamma)$$

$$\Downarrow$$

individual coordination payoffs

$$\Downarrow$$

$$n^* \in \Phi(n^*; \theta)$$

$$\Downarrow$$

$$G^*(z;\theta)$$

$$\Downarrow$$

$$C_N^{**}(z;\theta)$$

$$\Downarrow$$

$$s^*(z;\theta) = \arg\min_{s\in\{M,N,H\}} C_s^{**}(z;\theta).$$

The resulting organizational boundaries are therefore endogenous objects:

$$z_{MN}^* = z_{MN}^*(\theta), \qquad z_{NH}^* = z_{NH}^*(\theta).$$

This structure separates three logically distinct stages of the theory:

$$\boxed{\text{network formation} \to \text{coordination cost} \to \text{organizational choice.}}$$

That separation is useful both theoretically and empirically. It prevents the organizational-boundary results from being interpreted as primitive assumptions about firm structure and makes clear that the boundaries arise from equilibrium coordination technology.

The next appendix develops the network-formation problem in greater detail, including stability, sorting, increasing returns, and the conditions under which equilibrium concentration emerges.

B Network Formation and Equilibrium Stability

This appendix develops the network-formation component of the model in greater detail. The purpose is to establish formally how individual participation decisions generate an endogenous coordination network, how increasing returns in coordination capacity affect equilibrium structure, and under what conditions concentration among coordination providers emerges.

The analysis proceeds in five steps. First, we define the coordination network and the induced participation game. Second, we characterize individual best responses and symmetric equilibria. Third, we analyze local stability and the role of the scale elasticity γ. Fourth, we allow coordination providers to differ in productivity and characterize sorting and concentration. Finally, we establish the concentration result used in the main text.

B.1 B.1. The Coordination Network

Let

$$\mathcal{I} = [0, 1]$$

denote the continuum of productive agents and let

$$\mathcal{J} = \{1, \dots, J\}$$

denote the set of potential coordination providers.

A coordination network is a measurable assignment

$$g : \mathcal{I} \to \mathcal{J} \cup \{0\},$$

where $g(i) = j$ indicates that agent i coordinates through provider j, while $g(i) = 0$ denotes decentralized market coordination.

The induced participation measure of provider j is

$$n_j(g) = \mu\left(\{i \in \mathcal{I} : g(i) = j\}\right).$$

The vector

$$n(g) = (n_1(g), \ldots, n_J(g))$$

belongs to the simplex

$$\mathcal{N} = \left\{ n \in [0,1]^J : \sum_{j=1}^{J} n_j \leq 1 \right\}.$$

The aggregate scale of the coordination network is

$$N(g) = \sum_{j=1}^{J} n_j(g).$$

The network is therefore endogenous in two dimensions.

First, the set of participating agents is endogenous. Second, the allocation of participating agents across providers is endogenous.

B.2 B.2. Provider Capacity

Provider j has productivity parameter

$$a_j > 0.$$

Given participation n_j, its coordination capacity is

$$K_j(n_j) = A a_j n_j^{\gamma},$$

where

$$A > 0, \qquad \gamma > 0.$$

The marginal coordination capacity generated by an additional participant is

$$K_j'(n_j) = A a_j \gamma n_j^{\gamma-1}.$$

The elasticity of coordination capacity with respect to provider scale is

$$\frac{n_j K_j'(n_j)}{K_j(n_j)} = \gamma.$$

Thus the three cases

$$\gamma < 1, \qquad \gamma = 1, \qquad \gamma > 1$$

correspond respectively to decreasing, constant, and increasing returns to provider scale. This distinction is central to the network-formation problem.

B.3 B.3. Participation Payoffs

Let an agent of type ω obtain gross coordination benefit

$$B(\omega, K),$$

where

$$B_K(\omega, K) > 0.$$

If the agent participates in provider j, its payoff is

$$u_j(\omega, n_j) = B\left(\omega, A a_j n_j^{\gamma}\right) - c_j(\omega) - \tau(P)\mathbf{1}\{j \notin \Pi_F(i)\},$$

where $\Pi_F(i)$ denotes the ownership unit containing agent i.

For notational simplicity, define the effective participation cost

$$\widetilde{c}_j(\omega; P) = c_j(\omega) + \tau(P)\mathbf{1}\{j \notin \Pi_F(i)\}.$$

Then

$$u_j(\omega, n_j) = B\left(\omega, Aa_j n_j^{\gamma}\right) - \widetilde{c}_j(\omega; P).$$

The market alternative has payoff

$$u_0(\omega) = u_M(\omega).$$

An agent chooses provider j whenever

$$u_j(\omega, n_j) \geq \max\left\{u_0(\omega), u_k(\omega, n_k) : k \neq j\right\}.$$

B.4 B.4. Best-Response Correspondence

For each provider j, define the best-response set

$$BR_j(n; \theta) = \left\{\omega : u_j(\omega, n_j) \geq \max_{k \in \mathcal{J} \cup \{0\}} u_k(\omega, n_k)\right\}.$$

A participation allocation is an equilibrium if

$$n_j = \mu(BR_j(n; \theta))$$

for every provider j, up to sets of measure zero associated with indifference.

Define the aggregate best-response operator

$$T_j(n; \theta) = \mu(BR_j(n; \theta)).$$

Then a network-formation equilibrium satisfies

$$n^* = T(n^*; \theta).$$

The fixed-point representation makes the source of strategic feedback transparent.

An increase in n_j raises K_j according to

$$\frac{\partial K_j}{\partial n_j} = A a_j \gamma n_j^{\gamma-1},$$

which can raise the payoff to joining provider j, thereby increasing $T_j(n)$ and potentially generating further participation.

This feedback is positive whenever

$$B_K > 0.$$

Its strength depends on γ.

B.5 B.5. Symmetric Provider Environment

We first consider the benchmark in which all providers are identical:

$$a_j = a, \qquad c_j(\omega) = c(\omega)$$

for all j.

Suppose further that a symmetric equilibrium has

$$n_1 = \cdots = n_J = n.$$

The participation condition for a representative agent can then be written as

$$B(\omega, Aan^\gamma) - c(\omega) \geq u_0(\omega).$$

Define the participation surplus

$$V(\omega, n) = B(\omega, Aan^{\gamma}) - c(\omega) - u_0(\omega).$$

An interior symmetric equilibrium satisfies

$$V(\omega, n) = 0$$

for the marginal participating type.

If participation is summarized by a scalar cutoff $\bar{\omega}$, then

$$n = 1 - F(\bar{\omega})$$

and the marginal participation condition becomes

$$B\left(\bar{\omega}, Aa[1 - F(\bar{\omega})]^{\gamma}\right) - c(\bar{\omega}) = u_0(\bar{\omega}).$$

This equation provides the basic equilibrium condition used below.

B.6 B.6. A Scalar Network-Participation Formulation

For many applications it is useful to collapse the heterogeneous-agent problem into an aggregate participation equation.

Let

$$V(n; \theta)$$

denote the expected net payoff from participation at network scale n.

An interior equilibrium satisfies

$$V(n; \theta) = 0.$$

Suppose that

$$V_n(n;\theta)$$

exists.

Then local comparative statics follow from

$$\frac{dn^*}{d\theta_k} = -\frac{V_{\theta_k}(n^*;\theta)}{V_n(n^*;\theta)}.$$

The denominator is economically important.

If

$$V_n < 0,$$

the participation feedback is locally stabilizing.

If

$$V_n > 0,$$

the participation feedback is locally destabilizing.

The latter case is possible because a larger network raises the value of joining the network.

B.7 B.7. Local Stability

Consider the adjustment process

$$n_{t+1} = T(n_t;\theta).$$

An equilibrium n^* is locally stable under this adjustment process if

$$|T_n(n^*;\theta)| < 1.$$

It is locally unstable if

$$|T_n(n^*;\theta)| > 1.$$

In the scalar case, this follows from linearization around the fixed point:

$$n_{t+1} - n^* = T_n(n^*;\theta)(n_t - n^*) + o(|n_t - n^*|).$$

Hence deviations shrink when

$$|T_n| < 1$$

and grow when

$$|T_n| > 1.$$

The stability condition should not be confused with existence. An equilibrium may satisfy

$$n^* = T(n^*)$$

while failing the local stability condition.

B.8 B.8. The Role of the Scale Elasticity

The derivative of provider capacity is

$$K_j'(n_j) = Aa_j\gamma n_j^{\gamma-1}.$$

Consequently,

$$\frac{\partial u_j}{\partial n_j} = B_K Aa_j\gamma n_j^{\gamma-1}.$$

The behavior of this derivative near zero depends qualitatively on γ.

If

$$0 < \gamma < 1,$$

then

$$n_j^{\gamma-1} \to \infty \qquad \text{as } n_j \to 0^+.$$

If

$$\gamma = 1,$$

then

$$K_j'(n_j) = Aa_j.$$

If

$$\gamma > 1,$$

then

$$n_j^{\gamma-1} \to 0 \qquad \text{as } n_j \to 0^+.$$

Thus increasing returns do not automatically imply that a small provider grows rapidly from an arbitrarily small base. Indeed, when $\gamma > 1$, the marginal coordination capacity of a provider approaches zero near zero scale.

The economically relevant implication is instead that the average coordination capacity

$$\frac{K_j(n_j)}{n_j} = Aa_j n_j^{\gamma-1}$$

is increasing in n_j when

$$\gamma > 1.$$

This creates a coordination advantage for larger providers.

B.9 B.9. Increasing Returns and Multiple Equilibria

Consider the scalar participation equation

$$n = T(n).$$

Suppose that T is continuously differentiable and increasing.

If

$$T_n(n) > 1$$

over an interval containing a fixed point, then the fixed point is locally unstable.

Multiple equilibria can arise when the best-response curve intersects the 45-degree line more than once.

In particular, suppose there exist

$$0 < n_L < n_M < n_H < 1$$

such that

$$T(n_L) = n_L, \qquad T(n_M) = n_M, \qquad T(n_H) = n_H,$$

with

$$|T_n(n_L)| < 1, \qquad |T_n(n_M)| > 1, \qquad |T_n(n_H)| < 1.$$

Then the low- and high-participation equilibria are locally stable while the middle equilibrium is unstable.

This provides the formal basis for threshold effects discussed in Section 8.7.

The model therefore admits a coordination-based interpretation of multiple organizational equilibria: sufficiently strong network feedback can make participation self-reinforcing.

B.10 B.10. Provider Heterogeneity

We now allow providers to differ in coordination productivity:

$$a_j \neq a_k$$

for some j, k.

Suppose that participation costs are otherwise identical. Provider j gives type ω payoff

$$u_j(\omega, n_j) = B(\omega, A a_j n_j^{\gamma}) - c(\omega).$$

For any fixed participation levels $n_j = n_k = n$, if

$$a_j > a_k,$$

then

$$u_j(\omega, n) > u_k(\omega, n)$$

for all types with

$$B_K(\omega, K) > 0.$$

Thus provider productivity directly affects participation incentives.

However, equilibrium provider size need not be ordered solely by a_j because provider size itself affects coordination capacity.

The equilibrium comparison is therefore

$$a_j n_j^{\gamma} \quad \text{versus} \quad a_k n_k^{\gamma},$$

rather than a_j versus a_k alone.

This distinction becomes important for the concentration result.

B.11 B.11. Pairwise Provider Comparison

Suppose two providers j and k serve agents of the same type ω.

Provider j is preferred whenever

$$A a_j n_j^{\gamma} - c_j(\omega) \geq A a_k n_k^{\gamma} - c_k(\omega)$$

under a linear coordination-benefit specification.

More generally, because $B_K > 0$,

$$A a_j n_j^{\gamma} \geq A a_k n_k^{\gamma}$$

implies

$$u_j(\omega, n_j) - c_j(\omega) \geq u_k(\omega, n_k) - c_k(\omega)$$

when participation costs are equal.

Taking logarithms of the capacity terms gives

$$\log a_j + \gamma \log n_j \geq \log a_k + \gamma \log n_k.$$

Hence

$$\log \frac{n_j}{n_k} \geq -\frac{1}{\gamma} \log \frac{a_j}{a_k}$$

is the relevant scale-productivity tradeoff at equal effective capacity.

The equation also shows why provider productivity and provider scale cannot be identified separately from equilibrium capacity without additional information.

B.12 B.12. Concentration Under Increasing Returns

The central concentration mechanism can be seen directly from the average capacity per participant:

$$\bar{K}_j(n_j) = \frac{K_j(n_j)}{n_j} = A a_j n_j^{\gamma-1}.$$

Differentiating gives

$$\frac{\partial \bar{K}_j}{\partial n_j} = A a_j (\gamma - 1) n_j^{\gamma-2}.$$

Therefore,

$$\frac{\partial \bar{K}_j}{\partial n_j} > 0 \quad \iff \quad \gamma > 1.$$

This yields the fundamental concentration force.

Proposition 10 (Increasing returns create a concentration force)**.** *Suppose* $\gamma > 1$ *and* $a_j = a_k = a$ *for two coordination providers. Then, holding total participation*

$$n_j + n_k = \bar{n}$$

fixed, total coordination capacity

$$K_j(n_j) + K_k(n_k)$$

is strictly increased by moving participation toward the provider with the larger participation share, provided the initial allocation is sufficiently unequal.

Proof. With equal productivity,

$$K_j(n_j) + K_k(n_k) = Aa\left(n_j^\gamma + n_k^\gamma\right).$$

Because $\gamma > 1$, the function

$$x^\gamma$$

is strictly convex on $\mathbb{R}_+$.

Holding

$$n_j + n_k = \bar{n}$$

fixed, strict convexity implies that the sum

$$n_j^\gamma + n_k^\gamma$$

is increasing as the allocation becomes more unequal in the sense of a mean-preserving spread.

Thus concentrating participation on one provider increases total coordination capacity relative to a sufficiently balanced allocation. □

The proposition establishes a technological concentration force. It does not, by itself, establish that equilibrium is fully concentrated.

Entry costs, heterogeneous provider productivity, congestion, multi-homing, or other forces can counteract this force.

B.13 B.13. Symmetric Splitting Versus Concentration

The preceding result can be made explicit.

Suppose total participation is $\bar{n}$ and there are J identical providers.

Under equal splitting,

$$n_j = \frac{\bar{n}}{J}.$$

Total capacity is

$$K_{\text{split}} = JAa\left(\frac{\bar{n}}{J}\right)^{\gamma} = Aa\bar{n}^{\gamma}J^{1-\gamma}.$$

Under complete concentration,

$$n_1 = \bar{n}, \qquad n_j = 0 \quad (j \neq 1),$$

and total capacity is

$$K_{\text{conc}} = Aa\bar{n}^{\gamma}.$$

Therefore,

$$\frac{K_{\text{conc}}}{K_{\text{split}}} = J^{\gamma-1}.$$

Hence

$$K_{\text{conc}} > K_{\text{split}}$$

if and only if

$$\gamma > 1.$$

When

$$\gamma = 1,$$

the two allocations generate identical total capacity.

When

$$0 < \gamma < 1,$$

equal splitting generates greater aggregate capacity.

This is the cleanest formal statement of the scale-elasticity mechanism.

B.14 B.14. Concentration and Equilibrium Selection

The technological concentration force does not uniquely determine equilibrium selection.

When

$$\gamma > 1,$$

the production of coordination capacity is convex in provider scale. The equilibrium may therefore depend on initial conditions, entry, expectations, or other selection mechanisms.

To formalize this point, let

$$\mathcal{E}(\theta)$$

denote the set of network-formation equilibria.

The equilibrium correspondence may satisfy

$$|\mathcal{E}(\theta)| > 1.$$

The main-text concentration result should therefore be interpreted in one of two ways.

First, it may characterize the concentration of the efficient network allocation.

Second, under an explicitly specified adjustment process, it may characterize the stable equilibrium selected by that process.

The distinction is important because efficiency and dynamic stability need not coincide.

B.15 B.15. Efficient Network Allocation

For a fixed set of participating agents and fixed total participation $\bar{n}$, consider the planner's problem

$$\max_{\{n_j\}_{j=1}^J} \sum_{j=1}^J A a_j n_j^\gamma$$

subject to

$$\sum_{j=1}^J n_j = \bar{n}, \qquad n_j \geq 0.$$

The Lagrangian is

$$\mathcal{L} = \sum_{j=1}^J A a_j n_j^\gamma - \lambda \left(\sum_{j=1}^J n_j - \bar{n} \right).$$

For an interior solution,

$$A a_j \gamma n_j^{\gamma-1} = \lambda.$$

Hence

$$n_j = \left(\frac{\lambda}{Aa_j\gamma}\right)^{1/(\gamma-1)}$$

when $\gamma \neq 1$.

This expression demonstrates that the curvature of the coordination technology determines the shape of the efficient allocation.

For

$$0 < \gamma < 1,$$

the planner's objective is concave and the interior allocation is well behaved.

For

$$\gamma = 1,$$

the objective is linear and the allocation depends directly on relative productivities.

For

$$\gamma > 1,$$

the objective is convex, so an interior allocation cannot generally be the global maximum.

The global optimum is therefore attained at a boundary of the simplex, subject to provider productivity.

B.16 B.16. Efficient Concentration Theorem

The preceding observation yields the following result.

[Efficient concentration under increasing returns] Suppose that

$$\gamma > 1,$$

coordination benefits are increasing in total coordination capacity, and provider participation costs do not offset the convexity of the coordination technology.

For a fixed total participation level $\bar{n}$, an efficient allocation of participation assigns all participation to a provider with maximal coordination productivity

$$a_{\max} = \max_{j \in \mathcal{J}} a_j.$$

If the maximal productivity provider is unique, the efficient allocation is unique up to null sets of participants.

Proof. For any feasible allocation,

$$\sum_{j=1}^{J} A a_j n_j^{\gamma} \leq A a_{\max} \sum_{j=1}^{J} n_j^{\gamma}.$$

Because $\gamma > 1$,

$$\sum_{j=1}^{J} n_j^{\gamma} \leq \left(\sum_{j=1}^{J} n_j \right)^{\gamma} = \bar{n}^{\gamma},$$

with equality if and only if all positive participation is assigned to a single provider.

Therefore,

$$\sum_{j=1}^{J} A a_j n_j^{\gamma} \leq A a_{\max} \bar{n}^{\gamma}.$$

The upper bound is attained by assigning all participation to a provider with $a_j = a_{\max}$.

If the maximal productivity provider is unique, the maximizing allocation is unique. □

The theorem concerns the efficient allocation conditional on participation. It does not imply that a decentralized equilibrium necessarily selects the efficient allocation.

That distinction is essential when applying the result to observed market structure.

B.17 B.17. Why the Theorem Does Not Imply Monopoly

The concentration theorem should not be interpreted as a general monopoly theorem.

Several mechanisms can prevent full concentration.

First, provider-specific participation costs may satisfy

$$c_j(\omega) \neq c_k(\omega).$$

Second, portability costs may vary across ownership boundaries.

Third, congestion can introduce a term

$$-\chi(n_j)$$

into participation payoffs, with

$$\chi'(n_j) > 0.$$

Fourth, agents may multi-home.

Fifth, providers may face capacity constraints.

Sixth, entry may respond endogenously to provider profitability.

The relevant equilibrium condition is therefore

$$B(\omega, Aa_j n_j^{\gamma}) - c_j(\omega) - \tau(P) - \chi(n_j)$$

rather than coordination capacity alone.

The baseline concentration theorem identifies a force toward concentration; it does not assert that all countervailing forces are absent.

B.18 B.18. Network Concentration Versus Ownership Concentration

Let

$$H_C = \sum_{j=1}^{J} r_j^2$$

denote concentration in coordination provision, where

$$r_j = \frac{n_j}{\sum_k n_k}$$

in the simplest participation-based measure.

Let

$$H_F = \sum_m s_m^2$$

denote concentration in productive ownership.

The network model places no identity restriction between these two distributions.

In particular, it is possible to have

$$H_C > H_F$$

or

$$H_C < H_F.$$

The economically important case generated by increasing returns is

$$H_C \uparrow$$

while productive ownership becomes more dispersed.

Thus the concentration result is fundamentally a statement about the coordination layer of the economy.

B.19 B.19. Summary

The network-formation analysis establishes five results.

First, the coordination network can be represented as a fixed point of aggregate participation responses.

Second, existence does not imply uniqueness or stability.

Third, the elasticity

$$\gamma$$

determines the curvature of the coordination technology and therefore the strength and direction of scale effects.

Fourth, when

$$\gamma > 1,$$

coordination capacity is convex in provider participation, creating a force toward concentration.

Fifth, with heterogeneous provider productivity, the efficient allocation concentrates participation on the most productive provider, conditional on fixed aggregate participation and absent sufficiently strong countervailing costs.

The next part of this appendix extends the analysis from participation to explicit graph formation and derives the network-level stability and concentration conditions used in the main-text organizational-boundary theorem.

B.20 B.20. Explicit Graph Formation

The preceding analysis treated provider participation as the sufficient statistic for network formation. We now introduce the underlying graph explicitly.

Let

$$\mathcal{V} = \mathcal{I} \cup \mathcal{J}$$

denote the set of productive agents and coordination providers.

For each productive agent $i \in \mathcal{I}$ and provider $j \in \mathcal{J}$, define the binary link variable

$$g_{ij} \in \{0, 1\},$$

where

$$g_{ij} = 1$$

indicates that agent i maintains an active coordination relationship with provider j.

The network is therefore represented by the bipartite adjacency matrix

$$G = (g_{ij})_{i \in \mathcal{I}, j \in \mathcal{J}}.$$

The provider degree is

$$d_j(G) = \int_{\mathcal{I}} g_{ij} \, d\mu(i),$$

and, under the normalization used throughout the paper,

$$d_j(G) = n_j(G).$$

Thus the provider-scale variable used in the main model is the degree of the corresponding coordination node in the underlying bipartite graph.

The degree of productive agent i is

$$d_i(G) = \sum_{j=1}^{J} g_{ij}.$$

This permits the model to distinguish single-homing from multi-homing.

Under single-homing,

$$d_i(G) \in \{0, 1\},$$

whereas under multi-homing,

$$d_i(G) \geq 1$$

may hold for multiple providers.

B.21 B.21. Link-Specific Coordination

Provider-level scale is not the only determinant of coordination quality. Let

$$q_{ij} \geq 0$$

denote the effective quality of the relationship between agent i and provider j.

The baseline model assumes that

$$q_{ij} = q_j(n_j),$$

where

$$q_j(n_j) = A a_j n_j^{\gamma}.$$

The graph formulation therefore nests the provider-scale formulation.

More generally, we may allow

$$q_{ij} = Aa_j n_j^{\gamma} \ell_{ij},$$

where

$$\ell_{ij} > 0$$

is a relationship-specific productivity term.

The normalization

$$\ell_{ij} = 1$$

recovers the baseline model.

The advantage of the graph formulation is that it allows the theory to separate aggregate scale effects from relationship-specific matching effects.

B.22 B.22. Link Formation Payoffs

Given a network G, define the payoff to agent i from maintaining link (i, j) as

$$U_{ij}(G) = B\left(\omega_i, q_{ij}(G)\right) - c_{ij} - \tau_{ij}(P) - \kappa_{ij}(G),$$

where

$$c_{ij}$$

is the direct relationship cost,

$$\tau_{ij}(P)$$

is the cost of coordinating across organizational boundaries, and

$$\kappa_{ij}(G)$$

captures congestion, attention, or relationship-maintenance costs induced by the surrounding network.

The last term is not required for the baseline results. Setting

$$\kappa_{ij}(G) = 0$$

returns the pure scale-economies model.

The payoff to maintaining no relationship with provider j is normalized to zero relative to the relevant outside option.

A link is individually profitable whenever

$$U_{ij}(G) \geq 0.$$

B.23 B.23. Bilateral Link Formation

In the more general formulation, coordination relationships may require participation by both sides.

Let

$$U_{ij}^{i}(G)$$

denote the payoff to agent i from link (i, j) and

$$U_{ij}^{j}(G)$$

the corresponding payoff to provider j.

A link is formed only when both parties weakly prefer formation:

$$U^i_{ij}(G) \geq 0$$

and

$$U^j_{ij}(G) \geq 0.$$

A strict bilateral surplus condition is

$$U^i_{ij}(G) > 0, \qquad U^j_{ij}(G) > 0.$$

This formulation is useful because a social-media or platform relationship need not be technologically identical to an employment relationship.

The productive agent may benefit from access to coordination capacity while the provider benefits from the scale, information, reputation, or activity generated by the relationship.

B.24 B.24. Pairwise Stability

We use pairwise stability as the baseline static stability concept for the explicit graph.

[Pairwise stable network] A network G is pairwise stable if:

1. for every existing link (i, j) with $g_{ij} = 1$,

$$U^i_{ij}(G) \geq 0$$

and

$$U^j_{ij}(G) \geq 0;$$

and

2. for every absent link (i, j) with $g_{ij} = 0$, if

$$U^i_{ij}(G) > 0,$$

then

$$U^j_{ij}(G) < 0.$$

The first condition rules out profitable unilateral deletion of an existing relationship.

The second rules out mutually profitable additions.

Pairwise stability is deliberately weaker than global efficiency. A stable network may therefore contain inefficiently sparse or inefficiently concentrated relationships.

This distinction is central to the theory because organizational structure can be an equilibrium outcome rather than a planner-selected allocation.

B.25 B.25. Link Persistence

To distinguish persistent relationships from transitory interactions, let

$$s_{ij,t} \in \{0, 1\}$$

denote the persistence state of relationship (i, j) at date t.

A relationship is *persistent* if

$$s_{ij,t} = 1$$

for a nontrivial sequence of periods.

Let the transition probabilities be

$$\Pr(s_{ij,t+1} = 1 \mid s_{ij,t} = 1) = \rho_{ij}^{11}$$

and

$$\Pr(s_{ij,t+1} = 1 \mid s_{ij,t} = 0) = \rho_{ij}^{01}.$$

The persistence parameter is therefore

$$\rho_{ij}^{11} \in [0, 1].$$

High values of

$$\rho_{ij}^{11}$$

correspond to durable relationships, whereas low values correspond to high-turnover relationships.

The baseline static model can be interpreted as the limiting case in which the persistence state is fixed.

B.26 B.26. Persistent Network Capital

Define the stock of persistent network capital associated with agent i as

$$N_{i,t} = \sum_{j=1}^{J} s_{ij,t} g_{ij,t} v_{ij,t},$$

where

$$v_{ij,t} \geq 0$$

is the productive value of the relationship.

Network capital evolves according to

$$N_{i,t+1} = (1 - \delta_N) N_{i,t} + I_{i,t},$$

where

$$\delta_N \in [0, 1]$$

is the depreciation rate of relationship capital and $I_{i,t}$ denotes new network-capital investment.

The parameter δ_N is conceptually distinct from the portability parameter P.

Portability determines how readily productive relationships can be carried across organizational boundaries.

Persistence determines how long the underlying relationship remains valuable.

B.27 B.27. Network Density

For a finite approximation with I productive agents and J providers, define network density as

$$D(G) = \frac{\sum_{i=1}^{I} \sum_{j=1}^{J} g_{ij}}{IJ}.$$

Under a continuum of productive agents, the corresponding density is

$$D(G) = \frac{1}{J} \sum_{j=1}^{J} \int_{\mathcal{I}} g_{ij} \, d\mu(i).$$

Under single-homing,

$$D(G) = \frac{N(G)}{J}.$$

Thus density measures the prevalence of active coordination relationships, whereas provider concentration measures how those relationships are distributed across providers.

These are distinct objects.

B.28 B.28. Degree Distribution

Let

$$D_i(G) = \sum_{j=1}^{J} g_{ij}$$

denote the degree of productive agent i.

The empirical degree distribution is

$$F_D(k) = \mu\left(\{i : D_i(G) \leq k\}\right).$$

Similarly, the provider-side degree distribution is generated by

$$d_j(G) = \int_{\mathcal{I}} g_{ij}\, d\mu(i).$$

The theory therefore permits a distinction between:

network density,

provider concentration,

and

individual degree heterogeneity.

These quantities need not move together.

For example, an increase in portability may increase network density while simultaneously reducing provider concentration if productive agents can maintain relationships with multiple providers.

B.29 B.29. Persistent Versus Ephemeral Networks

Let

$$\bar{\rho} = \frac{\sum_{i,j} \rho_{ij}^{11} g_{ij}}{\sum_{i,j} g_{ij}}$$

denote the average persistence of active relationships.

We distinguish:

$$\bar{\rho} \to 1$$

as a persistent-network regime and

$$\bar{\rho} \to 0$$

as an ephemeral-network regime.

This distinction has direct organizational implications.

When relationships are persistent, organizational boundaries may internalize valuable relationship-specific capital.

When relationships are highly portable and ephemeral, the same productive relationships can be maintained without common ownership.

The model therefore does not equate network coordination with weak relationships. A network can be both highly persistent and organizationally external.

B.30 B.30. Network Formation with Portable Relationships

Let the cost of maintaining a relationship across organizational boundaries be

$$\tau(P),$$

with

$$\tau'(P) < 0.$$

The net value of an external relationship is therefore

$$U_{ij}^{\text{ext}} = B(\omega_i, q_{ij}) - c_{ij} - \tau(P) - \kappa_{ij}.$$

The value of an internal relationship is

$$U_{ij}^{\text{int}} = B(\omega_i, q_{ij}) - c_{ij} - \kappa_{ij}.$$

Hence

$$U_{ij}^{\text{int}} - U_{ij}^{\text{ext}} = \tau(P).$$

As portability rises,

$$P \uparrow \quad \Rightarrow \quad \tau(P) \downarrow,$$

and the organizational penalty associated with maintaining an external relationship falls.

This is the formal mechanism through which portability changes the organizational boundary without necessarily changing the underlying productive relationship.

B.31 B.31. Network Formation and Ownership

Let

$$o_{ij} \in \{0,1\}$$

indicate whether the relationship (i,j) is internal to the same ownership unit.

Define the ownership boundary indicator

$$b_{ij} = 1 - o_{ij}.$$

The total boundary-crossing relationship burden is

$$B(G) = \sum_{i,j} b_{ij} g_{ij} \tau(P).$$

The total organizational cost can then be written as

$$C(G) = C_{\text{coord}}(G) + C_{\text{boundary}}(G) + C_{\text{maint}}(G).$$

In particular,

$$C_{\text{boundary}}(G) = \tau(P) \sum_{i,j} b_{ij} g_{ij}.$$

Thus organizational integration can be interpreted as a choice to convert some boundary-crossing edges into internal edges.

This formulation provides the graph-theoretic representation of the organizational-boundary problem:

$$\boxed{\text{firm boundary} = \text{partition of the coordination graph.}}$$

B.32 B.32. Edge Rewiring

Consider a local rewiring operation that replaces an existing edge

$$(i, j)$$

with

$$(i, k).$$

The change in total surplus is

$$\Delta W_{i,j\to k} = U_{ik}(G') - U_{ij}(G) + \Delta W_{-i},$$

where

$$\Delta W_{-i}$$

captures the effect of the rewiring on other network members.

A stable network must rule out profitable bilateral rewiring operations.

This motivates a stronger stability notion.

[Pairwise-rewiring stability] A network G is pairwise-rewiring stable if no pair of agents can jointly replace an existing relationship or add a new relationship in a manner that strictly increases both agents' payoffs.

Pairwise-rewiring stability is useful for persistent networks because an agent may not merely choose whether to connect, but whether to substitute one relationship for another.

B.33 B.33. Stability of the Network Mapping

Let

$$\mathcal{T}(G; \theta)$$

denote the network-update operator generated by individual link decisions.

A network equilibrium satisfies

$$G^* = \mathcal{T}(G^*; \theta).$$

Consider a perturbation

$$G = G^* + \Delta G.$$

A first-order approximation gives

$$\mathcal{T}(G; \theta) - G^* = D_G \mathcal{T}(G^*; \theta) \Delta G + o(\|\Delta G\|).$$

Define the Jacobian

$$J_G = D_G \mathcal{T}(G^*; \theta).$$

The network is locally stable under the adjustment process if the spectral radius satisfies

$$\rho(J_G) < 1.$$

If

$$\rho(J_G) > 1,$$

the equilibrium is locally unstable.

This condition generalizes the scalar stability condition in Section B.7.

B.34 B.34. Scale Feedback in the Network Jacobian

The network Jacobian can be decomposed into a direct relationship component and a scale-feedback component:

$$J_G = J_{\text{direct}} + J_{\text{scale}}.$$

The scale component arises because

$$\frac{\partial q_{ij}}{\partial n_j} = A a_j \gamma n_j^{\gamma-1}.$$

Consequently,

$$J_{\text{scale}} \propto B_K A a_j \gamma n_j^{\gamma-1}.$$

The strength of network feedback therefore depends jointly on

$$A, \qquad a_j, \qquad \gamma, \qquad B_K,$$

and the equilibrium scale n_j.

This is important for identification.

Observed concentration alone cannot generally identify γ without additional information on the underlying coordination productivity and the marginal value of coordination.

B.35 B.35. Network Stability and Organizational Boundaries

Let the reduced-form network coordination cost be

$$C_N^{**}(z;\theta) = C_N(z, G^*(z;\theta);\theta).$$

If G^* is locally stable and differentiable, then

$$\frac{dC_N^{**}}{dz} = \frac{\partial C_N}{\partial z} + \frac{\partial C_N}{\partial G}\frac{dG^*}{dz}.$$

The second term captures the endogenous response of the network to the coordination requirement.

This term is absent from a model in which network structure is treated as fixed.

The organizational boundary therefore depends not only on direct coordination costs but also on how the network reorganizes when the scale or complexity of production changes.

This is the formal source of the distinction between a fixed organizational technology and an endogenous organizational boundary.

B.36 B.36. A Network-Boundary Stability Condition

Let

$$D_{MN}(z;\theta) = C_N^{**}(z;\theta) - C_M^*(z).$$

At a regular market–network boundary,

$$D_{MN}(z_{MN}^*;\theta) = 0$$

and

$$D_{MN,z} \neq 0.$$

Suppose the equilibrium network is locally stable. Then the boundary is locally stable under perturbations of network structure whenever

$$\left|\frac{\partial C_N}{\partial G}\frac{\partial G^*}{\partial z}\right|$$

is sufficiently small relative to

$$\left|\frac{\partial C_N}{\partial z} - \frac{dC_M^*}{dz}\right|.$$

The condition ensures that endogenous network adjustment does not overturn the local ordering of organizational costs.

Thus stability of the network and regularity of the organizational boundary are logically distinct but connected conditions.

B.37 B.37. Network Formation as a Partition Problem

Let

$$\Pi(G)$$

denote the partition of productive relationships induced by ownership.

The organizational problem can then be represented as

$$\Pi^* \in \arg\min_{\Pi} \left\{ C_{\text{coord}}(G) + C_{\text{boundary}}(G, \Pi; P) + C_{\text{maint}}(G) \right\}.$$

This representation clarifies the conceptual contribution of the framework.

The network is not itself the organization.

Rather,

$$\boxed{\text{network} = \text{productive relationship structure,}}$$

while

$$\boxed{\text{organization} = \text{ownership partition imposed on that structure.}}$$

The same productive network can therefore support different organizational boundaries.

This is the central distinction between network structure and firm structure in the model.

B.38 B.38. Limiting Cases

The explicit graph model nests several familiar organizational environments.

No network coordination. If

$$g_{ij} = 0$$

for all i, j, then all coordination occurs through the market.

Complete hierarchical integration. If every productive relationship is internal,

$$b_{ij} = 0$$

for every active edge, then

$$C_{\text{boundary}}(G) = 0.$$

Pure external network. If all active relationships cross ownership boundaries,

$$b_{ij} = 1$$

for all active edges, then

$$C_{\text{boundary}}(G) = \tau(P) \sum_{i,j} g_{ij}.$$

Perfect portability. If

$$\tau(P) \to 0,$$

the ownership status of an edge ceases to affect its direct coordination cost.

The organizational boundary can then become increasingly detached from the productive network itself.

Zero portability. If

$$\tau(P) \to \infty,$$

external coordination becomes prohibitively costly, pushing the model toward internal organization.

These limits connect the graph model to the canonical market–hierarchy framework while retaining the possibility of a genuinely independent network architecture.

B.39 B.39. Implications for Social and AI-Mediated Coordination

The graph formulation also clarifies the theoretical role of contemporary coordination technologies.

A technology need not change the physical production function directly in order to alter organizational structure.

It is sufficient for the technology to change one or more of

$$\tau(P), \qquad \rho_{ij}^{11}, \qquad \kappa_{ij}, \qquad q_{ij},$$

or the information available to agents when forming links.

Social-media systems can therefore affect organizational structure through relationship formation, persistence, reputation, and information diffusion.

AI systems can additionally affect

$$q_{ij}$$

by increasing the effective coordination capacity associated with a given network, while potentially reducing

$$\tau(P)$$

through automated communication, translation, search, and workflow coordination.

The theory therefore does not require AI to be inserted as an additional primitive solely because it is technologically novel. AI matters when it changes a structural parameter of the coordination problem.

This distinction is important for the interpretation of the theory: the fundamental object is not "AI" or "social media" per se, but the changing economics of coordination relationships.

B.40 B.40. Summary of the Graph Extension

The explicit graph formulation establishes the following hierarchy:

$$g_{ij} \to n_j \to K_j \to U_{ij} \to G^* \to C_N^{**} \to \Pi^*.$$

At the lowest level, g_{ij} represents an individual productive relationship.

Aggregation generates provider scale n_j, which determines coordination capacity K_j.

Capacity affects individual link payoffs and therefore feeds back into network formation.

The resulting equilibrium network G^* determines the cost of network coordination.

Finally, the ownership partition Π^* determines the organizational boundary.

The model therefore separates three objects that are often conflated:

$$\boxed{\text{relationship structure}, \qquad \text{coordination structure}, \qquad \text{ownership structure.}}$$

This separation is what allows the framework to describe economies in which productive coordination becomes increasingly networked without requiring the firm itself to disappear.

B.41 B.41. Existence of Pairwise-Stable Networks

We first establish existence of a stable network before imposing the increasing returns condition. This separation is useful because existence and concentration are logically distinct.

For the existence result, consider a finite approximation with

$$\mathcal{I} = \{1, \dots, I\}, \qquad \mathcal{J} = \{1, \dots, J\}.$$

Let

$$\mathcal{G} = \{0, 1\}^{I \times J}$$

denote the finite set of feasible bipartite networks.

For each potential link (i, j), let

$$\Delta^i_{ij}(G)$$

denote the change in agent i's payoff from adding the link when it is currently absent, and let

$$\Delta^j_{ij}(G)$$

denote the corresponding change in provider j's payoff.

For an existing link, the same notation denotes the payoff change from deleting it.

We impose the following regularity condition.

Assumption 8 (Finite-link regularity)**.** *For every network $G \in \mathcal{G}$ and every potential link (i, j), payoffs are finite and well defined. Link payoffs depend only on the finite network G and the primitives θ.*

Because $\mathcal{G}$ is finite, this assumption rules out pathological nonexistence arising from undefined or unbounded link payoffs.

Proposition 11 (Existence of a pairwise-stable network)**.** *Suppose Assumption 8 holds and the network formation process is represented by a finite potential game. Then at least one pairwise-stable network exists.*

Proof. A finite potential game possesses at least one pure-strategy Nash equilibrium because a potential function

$$\Phi : \mathcal{G} \to \mathbb{R}$$

attains a maximum on the finite set $\mathcal{G}$.

Let

$$G^* \in \arg\max_{G \in \mathcal{G}} \Phi(G).$$

Suppose that G^* were not pairwise stable. Then either an existing link could be profitably deleted or an absent link could be jointly added in a way that strictly increases the payoffs of the relevant parties.

By the potential-game property, such a profitable deviation would strictly increase $\Phi(G)$, contradicting the maximality of G^*.

Therefore G^* is pairwise stable. □

The proposition is deliberately stated for a potential-game environment. Without such a condition, finiteness alone does not guarantee pairwise stability: unilateral and bilateral incentives can cycle.

The potential-game assumption is therefore substantive rather than merely technical.

B.42 B.42. A Sufficient Potential

A convenient potential for the present model can be constructed when link surpluses are additively separable.

Suppose total network surplus is

$$\Phi(G) = \sum_{i=1}^{I} \sum_{j=1}^{J} g_{ij} \left[B_{ij}(G) - c_{ij} - \tau_{ij}(P)\right] - \mathcal{C}(G),$$

where $\mathcal{C}(G)$ collects congestion and other network-level costs.

If the marginal payoff of a link to each participant equals its marginal contribution to Φ, then the network formation game is an exact potential game.

This occurs, for example, when link-specific externalities are symmetric and the coordination benefit generated by a provider is shared according to a common surplus rule.

The assumption is restrictive, but useful because it provides a transparent sufficient condition for existence.

B.43 B.43. Continuum Limit

The finite-network result provides the foundation for the continuum model.

Let

$$I \to \infty$$

while the empirical distribution of agent types converges weakly to

$$F(\omega).$$

Suppose that link payoffs are bounded and continuous in aggregate degree.

A sequence of finite networks

$$\{G^I\}_{I=1}^{\infty}$$

generates an aggregate network measure

$$\mu_G$$

in the limit.

Under standard compactness and continuity conditions, any convergent subsequence whose deviation incentives vanish in the limit generates a pairwise-stable continuum network.

The continuum formulation used in the main text should therefore be interpreted as the limit of the finite-agent network game rather than as a primitive replacement for it.

B.44 B.44. Increasing Returns and Concentration

We now impose the key technological condition

$$\gamma > 1.$$

Let two providers j and k have participation levels

$$n_j > n_k > 0.$$

For simplicity, first assume

$$a_j = a_k = a$$

and identical participation costs.

The average coordination capacity of provider j is

$$\bar{K}_j = Aan_j^{\gamma-1},$$

while that of provider k is

$$\bar{K}_k = Aan_k^{\gamma-1}.$$

Since

$$\gamma - 1 > 0$$

and

$$n_j > n_k,$$

we have

$$\bar{K}_j > \bar{K}_k.$$

Thus the larger provider supplies greater coordination capacity per participant.

This is the local source of the concentration effect.

B.45 B.45. A Link-Surplus Condition

To characterize when this technological force translates into network concentration, define the marginal link surplus to provider j as

$$S_j(n_j, \omega) = B_K\left(\omega, Aa_j n_j^{\gamma}\right) Aa_j \gamma n_j^{\gamma-1} - c_j(\omega) - \tau_j(P) - \kappa_j'(n_j).$$

The first term is the coordination-capacity benefit generated by the marginal link.

The remaining terms capture marginal costs.

Define the net scale-feedback term

$$R_j(n_j, \omega) = B_K\left(\omega, Aa_j n_j^{\gamma}\right) Aa_j \gamma n_j^{\gamma-1} - \kappa_j'(n_j).$$

We say that provider j exhibits *net increasing network returns* on an interval $\mathcal{N}$ if

$$\frac{\partial R_j(n, \omega)}{\partial n} > 0$$

for all relevant (n, ω).

Notice that

$$\gamma > 1$$

alone does not guarantee net increasing network returns if congestion rises sufficiently rapidly.

This qualification will be maintained throughout.

B.46 B.46. A Concentration Lemma

[Local advantage of a larger provider] Suppose that providers j and k have equal productivity and equal direct participation costs. Suppose further that net network returns are increasing in provider scale. If

$$n_j > n_k,$$

then the marginal surplus from an additional relationship is greater at provider j than at provider k:

$$S_j(n_j, \omega) > S_k(n_k, \omega).$$

Proof. Equal productivity and equal direct costs imply that the difference in marginal surplus arises only from the scale-dependent component.

By assumption,

$$R_j(n, \omega)$$

is increasing in n. Since

$$n_j > n_k,$$

it follows that

$$R_j(n_j, \omega) > R_k(n_k, \omega).$$

Therefore,

$$S_j(n_j, \omega) > S_k(n_k, \omega).$$

□

The lemma formalizes the feedback mechanism that was previously described verbally.

Once one provider becomes sufficiently larger, the marginal value of assigning another relationship to that provider exceeds the corresponding value at a smaller provider.

B.47 B.47. Core-Periphery Networks

The previous result motivates a core-periphery characterization.

Let the provider set be partitioned into

$$\mathcal{C}$$

and

$$\mathcal{P},$$

where $\mathcal{C}$ denotes the core and $\mathcal{P}$ the periphery.

A network has a core-periphery structure if there exists

$$n_c > n_p$$

for all

$$c \in \mathcal{C}, \qquad p \in \mathcal{P},$$

and the marginal link surplus satisfies

$$S_c(n_c, \omega) > 0$$

while

$$S_p(n_p, \omega) \leq 0$$

for the marginal participating types.

The core therefore attracts additional relationships while peripheral providers cannot profitably expand at their current scale.

B.48 B.48. Core-Periphery Equilibrium

[Core-periphery equilibrium under net increasing returns] Suppose:

1. the network formation game admits a pairwise-stable equilibrium;
2. providers are sufficiently symmetric that differences in participation costs do not overturn scale effects;

3. net network returns are increasing over the relevant range;
4. $\gamma > 1$;
5. congestion costs are bounded so that the marginal surplus of a sufficiently large provider remains positive over the relevant expansion range.

Then any pairwise-stable equilibrium exhibiting sufficiently unequal provider sizes admits a core-periphery representation: providers above a threshold size constitute a core, while providers below the threshold cannot profitably attract marginal relationships from the core.

Proof. Let the threshold $\bar{n}$ satisfy

$$S(\bar{n}, \omega) = 0$$

for the relevant marginal type.

Because net network returns are increasing,

$$S_n(n, \omega) > 0.$$

Hence

$$n > \bar{n} \quad \Rightarrow \quad S(n, \omega) > 0,$$

while

$$n < \bar{n} \quad \Rightarrow \quad S(n, \omega) < 0.$$

Consider a stable equilibrium with unequal provider sizes.

Any provider with

$$n_j > \bar{n}$$

has positive marginal link surplus and therefore belongs to the core.

Any provider with

$$n_k < \bar{n}$$

has non-positive marginal surplus at its current scale.

By pairwise stability, a peripheral provider cannot induce a mutually profitable deviation that transfers a marginal relationship away from the core.

Hence the equilibrium admits a partition into a set of above-threshold core providers and below-threshold peripheral providers. □

The theorem establishes a core-periphery implication without asserting that there must be a unique core provider.

Full concentration requires additional conditions.

B.49 B.49. Full Concentration

We now state the stronger result.

Let

$$a_{(1)} = \max_j a_j$$

and suppose provider 1 is the unique productivity leader:

$$a_1 > a_j \qquad \forall j \neq 1.$$

Assume that the coordination benefit is sufficiently strong relative to provider-specific costs.

[Stable concentration under strong increasing returns] Suppose:

1. $\gamma > 1$;

2. provider 1 has strictly maximal productivity,

$$a_1 > a_j \quad \forall j \neq 1;$$

3. net network returns are increasing;

4. congestion does not overturn the productivity and scale advantage of provider 1;

5. agents single-home; and

6. for every peripheral provider $j \neq 1$, the gain from moving a marginal relationship from j to provider 1 is strictly positive.

Then every pairwise-stable equilibrium with positive aggregate participation assigns all participating relationships to provider 1.

Proof. Suppose, to the contrary, that a pairwise-stable equilibrium has

$$n_j > 0$$

for some

$$j \neq 1.$$

Since

$$a_1 > a_j$$

and $\gamma > 1$, provider 1 has both a productivity advantage and, under the stated scale condition, a coordination-capacity advantage once it has sufficient participation.

By assumption 6, transferring a marginal relationship from provider j to provider 1 strictly raises the participating agent's payoff and does not reduce the provider-side payoff.

Therefore the transfer is a profitable bilateral deviation.

This contradicts pairwise stability.

Hence

$$n_j = 0$$

for every

$$j \neq 1.$$

Thus all participating relationships are assigned to provider 1. $\square$

The theorem is intentionally stronger than the core-periphery theorem and correspondingly requires stronger assumptions.

Most importantly, increasing returns alone do not imply full concentration.

B.50 B.50. A More General Concentration Criterion

The preceding theorem can be stated without requiring identical costs.

Define the effective provider advantage

$$\Lambda_j(n_j, n_k, \omega) = B(\omega, Aa_j n_j^{\gamma}) - c_j(\omega) - \tau_j(P) - \kappa_j(n_j).$$

Provider j dominates provider k for type ω whenever

$$\Lambda_j(n_j, n_k, \omega) > \Lambda_k(n_k, n_j, \omega).$$

Define the dominance condition

$$\inf_{\omega \in \Omega} [\Lambda_1(n_1, n_j, \omega) - \Lambda_j(n_j, n_1, \omega)] > 0.$$

Then a sufficient condition for elimination of provider j from a stable network is

(B.50).

This formulation makes clear that concentration depends on the joint distribution of

$$a_j, \quad c_j, \quad \tau_j, \quad \kappa_j,$$

rather than on γ alone.

B.51 B.51. Portability and Ownership Partitions

We now turn to the second central result.

The productive graph G and the ownership partition Π are distinct objects.

Let total surplus associated with a productive graph be

$$W(G, \Pi; P) = W_N(G) - \tau(P)B(G, \Pi),$$

where

$$W_N(G)$$

contains all ownership-invariant productive benefits and costs, while

$$B(G, \Pi)$$

is the number or weighted value of productive relationships that cross organizational boundaries.

The organizational problem is

$$\Pi^*(G; P) \in \arg\max_{\Pi} W(G, \Pi; P).$$

The network-formation problem is

$$G^*(P) \in \arg\max_G W(G, \Pi^*(G; P); P)$$

in the planner formulation, or the corresponding equilibrium problem in the decentralized formulation.

B.52 B.52. Ownership-Invariance Condition

Suppose that portability affects only the cost of crossing organizational boundaries:

$$\frac{\partial W_N(G)}{\partial P} = 0.$$

Suppose further that the incentives to form productive relationships depend only on $W_N(G)$ and not on the ownership label Π.

Then portability affects the ownership partition but does not directly affect the productive graph.

This yields the following result.

Proposition 12 (Portability can change ownership without changing the productive network)**.** *Suppose that:*

1. *productive network surplus $W_N(G)$ is independent of portability;*
2. *portability enters payoffs only through the boundary-crossing cost $\tau(P)$;*
3. *link formation depends on productive surplus but not directly on ownership labels; and*
4. *the equilibrium productive graph is locally unique.*

Then, over any interval of P for which the network equilibrium remains locally unique,

$$\frac{dG^*}{dP} = 0,$$

while the optimal ownership partition satisfies

$$\frac{d\Pi^*}{dP} \neq 0$$

whenever changes in $\tau(P)$ alter the relative cost of internal and external coordination.

Proof. By assumptions 1–3, the first-order conditions governing productive link formation do not depend on P.

Therefore the network equilibrium satisfies

$$F(G^*) = 0$$

for a function F independent of P.

By assumption 4 and the implicit function theorem,

$$\frac{dG^*}{dP} = 0.$$

The ownership problem, however, depends on

$$\tau(P)B(G^*, \Pi).$$

Since

$$\tau'(P) < 0,$$

changes in P alter the relative cost of internal versus external relationships whenever both types of partitions are feasible.

Hence Π^* can change despite

$$G^*$$

remaining unchanged. $\square$

This proposition is particularly important for the theory because it separates a change in the organization of production from a change in the productive relationships themselves.

B.53 B.53. The Ownership-Rewiring Distinction

Define a *network-preserving organizational transformation* as a change from

$$\Pi$$

to

$$\Pi'$$

such that

$$G(\Pi') = G(\Pi).$$

The transformation changes only ownership labels.

Define a *network-rewiring transformation* as a change satisfying

$$G' \neq G.$$

These are fundamentally different margins.

Portability can therefore generate either

$$\Pi \to \Pi' \qquad \text{with} \qquad G' = G,$$

or

$$(G, \Pi) \to (G', \Pi').$$

The first is a pure organizational-boundary response.

The second is a technological response that changes productive relationships themselves.

The distinction allows the theory to generate an empirically testable prediction:

$$\boxed{\text{organizational change need not imply network rewiring.}}$$

B.54 B.54. The General Organizational-Boundary Theorem

We can now combine the network and ownership components.

Let

$$C_M^*(z)$$

denote optimized market coordination cost,

$$C_N^*(z, P)$$

optimized external-network coordination cost, and

$$C_F^*(z)$$

optimized internal coordination cost.

Define

$$\Delta_{MN}(z, P) = C_N^*(z, P) - C_M^*(z)$$

and

$$\Delta_{NF}(z, P) = C_F^*(z) - C_N^*(z, P).$$

The organizational regime is determined by the sign pattern of these differences.

[General organizational-boundary theorem] Suppose:

1. the productive network equilibrium $G^*(z, P)$ exists and is locally stable;
2. optimized coordination costs are continuously differentiable in (z, P);
3. each pairwise organizational-cost difference has a regular zero; that is,

   $$\Delta_{ab}(z^*, P^*) = 0$$

   implies

   $$\frac{\partial \Delta_{ab}}{\partial z} \neq 0;$$

   and

4. portability reduces the marginal cost of external coordination:

   $$\frac{\partial C_N^*}{\partial P} < 0.$$

Then locally the economy admits well-defined organizational boundaries separating market, network, and firm regimes.

Moreover, the network regime expands relative to the firm regime as portability increases whenever

$$\frac{\partial C_N^*}{\partial P} < 0$$

and

$$\frac{\partial C_F^*}{\partial P} = 0.$$

The boundaries satisfy

$$\frac{dz_{NF}^*}{dP} = -\frac{\partial \Delta_{NF}/\partial P}{\partial \Delta_{NF}/\partial z}.$$

Proof. Existence and local stability of G^* imply that optimized network costs can be represented locally as a differentiable reduced-form function.

Consider the network–firm boundary:

$$\Delta_{NF}(z, P) = 0.$$

By assumption 3,

$$\Delta_{NF,z} \neq 0.$$

The implicit function theorem therefore yields a locally unique function

$$z_{NF}^*(P)$$

satisfying

$$\Delta_{NF}(z_{NF}^*(P), P) = 0.$$

Differentiating gives

$$\Delta_{NF,z} \frac{dz_{NF}^*}{dP} + \Delta_{NF,P} = 0,$$

which implies

$$\frac{dz^*_{NF}}{dP} = -\frac{\Delta_{NF,P}}{\Delta_{NF,z}}.$$

Since

$$\Delta_{NF} = C^*_F - C^*_N,$$

and

$$C^*_{F,P} = 0,$$

we have

$$\Delta_{NF,P} = -C^*_{N,P} > 0.$$

Thus the direction of the boundary shift is determined by the sign of $\Delta_{NF,z}$.

An analogous argument applies to the market–network boundary.

Hence portability shifts the endogenous organizational boundaries whenever it changes the relative cost of network and firm coordination. □

B.55 B.55. The Main Structural Result

The preceding results can be summarized in a single structural chain:

$$\boxed{\gamma \to \text{network scale returns} \to G^* \to C^*_N \to \Pi^*}$$

while portability operates primarily through

$$\boxed{P \to \tau(P) \to \Pi^*}$$

when the ownership-invariance condition holds.

Thus the two technological dimensions have distinct theoretical roles.

The scale elasticity

$$\gamma$$

governs the internal structure of the coordination network.

Portability

$$P$$

governs the extent to which that network must be contained within a common ownership boundary.

This distinction is what permits the theory to generate organizational transformation without requiring the productive network itself to disappear or become vertically integrated.

B.56 B.56. Interpretation

The results above imply that three dimensions of economic organization must be distinguished.

First is the *topology of production*:

$$G^*.$$

Second is the *scale structure of coordination*:

$$\{n_j^*\}_{j=1}^J.$$

Third is the *ownership partition*:

$$\Pi^*.$$

Canonical theories of the firm largely ask when the third object should absorb the second.

The present framework instead allows all three to be determined separately.

In particular, technological progress can produce the configuration

$$G^*_{\text{old}} = G^*_{\text{new}}, \qquad n^*_{\text{old}} = n^*_{\text{new}}, \qquad \Pi^*_{\text{old}} \neq \Pi^*_{\text{new}},$$

when portability changes while productive coordination technology remains unchanged.

Alternatively, technological progress can produce

$$G^*_{\text{old}} \neq G^*_{\text{new}},$$

when it changes the incentives to form productive relationships.

The distinction is central to the theory's claim that the boundaries of the firm need not coincide with the boundaries of production.

B.57 B.57. Primitive Specification of Network Returns

We now specialize the coordination technology sufficiently to derive the network-formation results from primitive parameters.

The specialization is deliberately parsimonious. Let the coordination capacity of provider j be

$$K_j(n_j) = A a_j n_j^{\gamma}, \qquad A > 0, \qquad a_j > 0, \qquad \gamma > 0.$$

Let the productive value of coordination to an agent of type ω be linear in effective coordination capacity:

$$B(\omega, K) = \omega K, \qquad \omega > 0.$$

The gross value generated by a relationship with provider j is therefore

$$V_{ij}(n_j) = \omega_i A a_j n_j^{\gamma}.$$

Let the private cost of maintaining a relationship with provider j be

$$c_j + \tau_j(P) + \kappa_j,$$

where c_j is the provider-specific participation cost, $\tau_j(P)$ is the organizational-boundary cost, and κ_j is a relationship-maintenance or congestion cost.

For the primitive threshold result, define

$$m_j(P) = c_j + \tau_j(P) + \kappa_j.$$

The net value of participation for type ω_i is therefore

$$U_{ij}(n_j; P) = \omega_i A a_j n_j^{\gamma} - m_j(P).$$

The corresponding marginal value of increasing provider participation is

$$\frac{\partial U_{ij}}{\partial n_j} = \omega_i A a_j \gamma n_j^{\gamma-1}.$$

When

$$\gamma > 1,$$

the marginal value of provider scale is strictly increasing:

$$\frac{\partial^2 U_{ij}}{\partial n_j^2} = \omega_i A a_j \gamma(\gamma - 1) n_j^{\gamma-2} > 0.$$

Thus increasing returns are a primitive property of the coordination technology rather than a reduced-form assumption about link incentives.

B.58 B.58. The Critical-Mass Threshold

A provider is attractive to an agent whenever

$$U_{ij}(n_j; P) \geq 0.$$

Under the primitive specification,

$$\omega_i A a_j n_j^{\gamma} \geq m_j(P).$$

Define the critical participation level for provider j and agent type ω by

$$\boxed{\bar{n}_j(\omega, P) = \left[\frac{m_j(P)}{\omega A a_j}\right]^{1/\gamma}.}$$

Participation is then characterized by

$$U_{ij}(n_j; P) \geq 0 \quad \Longleftrightarrow \quad n_j \geq \bar{n}_j(\omega_i, P).$$

The threshold is decreasing in productivity:

$$\frac{\partial \bar{n}_j}{\partial a_j} < 0,$$

decreasing in agent type:

$$\frac{\partial \bar{n}_j}{\partial \omega} < 0,$$

and, because

$$\tau_j'(P) < 0,$$

decreasing in portability:

$$\frac{\partial \bar{n}_j}{\partial P} < 0.$$

More explicitly,

$$\frac{\partial \bar{n}_j}{\partial P} = \frac{1}{\gamma}\bar{n}_j \frac{\tau_j'(P)}{m_j(P)} < 0.$$

Thus portability lowers the critical mass required for external network coordination.

B.59 B.59. Why the Critical-Mass Result Requires Increasing Returns

The role of γ can be seen directly from the participation condition.

If

$$\gamma = 1,$$

then

$$U_{ij}(n_j; P) = \omega_i A a_j n_j - m_j(P),$$

and the marginal value of participation is constant:

$$\frac{\partial U_{ij}}{\partial n_j} = \omega_i A a_j.$$

There is no increasing-return feedback.

If

$$0 < \gamma < 1,$$

then

$$\frac{\partial^2 U_{ij}}{\partial n_j^2} < 0,$$

and the marginal value of additional participation falls with network size.

Only when

$$\gamma > 1$$

does the model generate a self-reinforcing scale mechanism:

$$n_j \uparrow \implies \frac{\partial U_{ij}}{\partial n_j} \uparrow .$$

The critical-mass interpretation therefore has a technological foundation.

B.60 B.60. A Primitive Critical-Mass Proposition

Proposition 13 (Primitive critical-mass condition)**.** *Under*

$$K_j(n_j) = Aa_j n_j^{\gamma}, \qquad B(\omega, K) = \omega K, \qquad \gamma > 1,$$

a provider j attracts a type-ω relationship if and only if

$$n_j \geq \left[\frac{c_j + \tau_j(P) + \kappa_j}{\omega A a_j}\right]^{1/\gamma}.$$

Moreover, the threshold is strictly decreasing in A, a_j, ω, and P whenever $\tau_j'(P) < 0$.

Proof. The participation condition is

$$\omega A a_j n_j^{\gamma} \geq c_j + \tau_j(P) + \kappa_j.$$

Since all terms are positive and $\gamma > 0$, taking the $1/\gamma$ power gives

$$n_j \geq \left[\frac{c_j + \tau_j(P) + \kappa_j}{\omega A a_j} \right]^{1/\gamma}.$$

Differentiation gives the stated comparative statics. □

B.61 B.61. Heterogeneous Agents and Provider Activation

The threshold result becomes more informative when agent types are heterogeneous.

Let the distribution of productive-agent types be

$$F(\omega),$$

with support

$$[\underline{\omega}, \overline{\omega}].$$

Given provider scale n_j, the marginal participating type satisfies

$$\omega_j^*(n_j, P) = \frac{m_j(P)}{A a_j n_j^\gamma}.$$

Agents with

$$\omega_i \geq \omega_j^*(n_j, P)$$

find participation weakly profitable.

Hence the mass of agents willing to participate with provider j is

$$N_j(n_j, P) = 1 - F\left(\frac{m_j(P)}{A a_j n_j^\gamma} \right).$$

An equilibrium provider scale satisfies

$$n_j = N_j(n_j, P).$$

Equation (B.61) is the primitive network formation fixed point.

It makes the scale feedback explicit:

$$n_j \uparrow \quad \Rightarrow \quad \omega_j^* \downarrow \quad \Rightarrow \quad N_j \uparrow .$$

The provider's scale attracts additional types, which further increases scale.

B.62 B.62. A Primitive Network Activation Threshold

Consider the highest-cost type willing to participate, $\underline{\omega}$.

Define

$$\bar{n}_j^{\text{all}}(P) = \left[\frac{m_j(P)}{\underline{\omega} A a_j}\right]^{1/\gamma}.$$

If

$$n_j \geq \bar{n}_j^{\text{all}}(P),$$

then every type in the support of F weakly prefers participation with provider j.

Conversely, define

$$\bar{n}_j^{\text{top}}(P) = \left[\frac{m_j(P)}{\overline{\omega} A a_j}\right]^{1/\gamma}.$$

For

$$n_j < \bar{n}_j^{\text{top}}(P),$$

even the highest-value type does not find participation profitable.

Thus the network possesses an explicit activation region:

$$n_j < \bar{n}_j^{\text{top}} \quad \Rightarrow \quad \text{no voluntary activation,}$$

and an expansion region above the relevant critical mass.

B.63 B.63. Provider Productivity and Critical Mass

For two providers j and k, suppose

$$a_j > a_k$$

and

$$m_j(P) = m_k(P) = m.$$

Then

$$\frac{\bar{n}_j(\omega, P)}{\bar{n}_k(\omega, P)} = \left(\frac{a_k}{a_j}\right)^{1/\gamma} < 1.$$

The more productive provider therefore requires a smaller installed base to become viable.

This is the primitive source of provider selection.

Importantly, it does not require that the more productive provider initially have a larger network.

A provider with a productivity advantage may remain inactive if it begins below critical mass.

This generates the possibility of multiple stable network configurations.

B.64 B.64. Coordination Technology and Multiple Equilibria

Consider a symmetric provider with productivity a and cost m.

An equilibrium scale solves

$$n = 1 - F\left(\frac{m}{Aan^{\gamma}}\right).$$

Define

$$\Psi(n) = 1 - F\left(\frac{m}{Aan^{\gamma}}\right) - n.$$

An equilibrium satisfies

$$\Psi(n) = 0.$$

The derivative is

$$\Psi'(n) = f\left(\frac{m}{Aan^{\gamma}}\right)\frac{m\gamma}{Aan^{\gamma+1}} - 1.$$

Consequently, multiple fixed points can arise when the scale-feedback term is sufficiently strong:

$$f\left(\frac{m}{Aan^{\gamma}}\right)\frac{m\gamma}{Aan^{\gamma+1}} > 1$$

over some interval.

Thus increasing returns can generate:

$$\text{low-scale equilibrium}, \qquad \text{unstable threshold}, \qquad \text{high-scale equilibrium}.$$

This is the precise sense in which the model generates a network "takeoff" mechanism.

B.65 B.65. Explicit Threshold under a Uniform Type Distribution

For an explicit closed-form illustration, suppose

$$\omega \sim U[\underline{\omega}, \overline{\omega}].$$

Then

$$F(\omega) = \frac{\omega - \underline{\omega}}{\overline{\omega} - \underline{\omega}}.$$

The provider-demand equation becomes

$$n_j = \frac{\overline{\omega} - m_j(P)/(Aa_j n_j^{\gamma})}{\overline{\omega} - \underline{\omega}}.$$

Multiplying through gives

$$(\overline{\omega} - \underline{\omega})n_j = \overline{\omega} - \frac{m_j(P)}{Aa_j n_j^{\gamma}}.$$

Equivalently,

$$Aa_j(\overline{\omega} - \underline{\omega})n_j^{\gamma+1} - Aa_j\overline{\omega}n_j^{\gamma} + m_j(P) = 0.$$

This polynomial characterizes equilibrium provider scale.

Although a closed-form solution is generally unavailable for arbitrary γ, the critical-mass thresholds remain explicit through equation (B.58).

The distinction is useful: the *participation threshold* has a closed form even when the aggregate equilibrium scale does not.

B.66 B.66. A Primitive Core-Periphery Condition

We can now state a primitive sufficient condition for core-periphery formation.

Let providers be ordered by effective productivity net of participation cost:

$$\chi_j(P) = \frac{Aa_j}{m_j(P)}.$$

A larger $\chi_j(P)$ means that provider j converts a unit of network scale into coordination value more efficiently relative to its participation cost.

Suppose

$$\chi_1(P) > \chi_2(P) \geq \cdots \geq \chi_J(P).$$

Then

$$\bar{n}_1(\omega, P) < \bar{n}_2(\omega, P) \leq \cdots \leq \bar{n}_J(\omega, P).$$

Provider 1 therefore has the smallest critical mass.

B.67 B.67. Core-Periphery Theorem in Primitive Parameters

[Primitive core-periphery condition] Suppose:

1. $$K_j(n_j) = Aa_j n_j^{\gamma}, \qquad A > 0, \qquad \gamma > 1;$$

2. $$B(\omega, K) = \omega K;$$

3. agents single-home;

4. $$\chi_1(P) > \chi_j(P) \qquad \forall j \neq 1;$$

5. the provider-demand fixed point exists; and

6. the high-scale equilibrium of provider 1 is locally stable.

Then any equilibrium in which provider 1 crosses its critical-mass threshold satisfies

$$n_1 > \bar{n}_1(\omega, P)$$

for the relevant marginal type, while providers satisfying

$$n_j < \bar{n}_j(\omega, P)$$

cannot attract that marginal type.

Consequently, equilibrium network structure is core-periphery: provider 1 constitutes the core whenever it is activated, while providers that remain below their respective critical masses constitute the periphery.

Proof. From the primitive participation condition,

$$n_j \geq \bar{n}_j(\omega, P)$$

is necessary and sufficient for a type-ω relationship to be profitable.

Since

$$\chi_1(P) > \chi_j(P),$$

we have

$$\bar{n}_1(\omega, P) < \bar{n}_j(\omega, P)$$

for every $j \neq 1$.

Hence provider 1 requires the smallest installed base to attract the relevant type.

If provider 1 crosses its critical mass, increasing returns imply that additional scale lowers the marginal type required for participation:

$$\frac{\partial \omega_1^*}{\partial n_1} = -\frac{\gamma m_1(P)}{A a_1 n_1^{\gamma+1}} < 0.$$

The activated provider therefore attracts a weakly larger set of types as scale expands.

A provider j below its critical mass cannot profitably attract the relevant marginal type. Hence the activated provider forms the core and inactive or subcritical providers form the periphery. □

B.68 B.68. Full Concentration as a Stronger Result

Core-periphery structure does not imply that all relationships must belong to one provider.

To obtain full concentration, we require a stronger condition.

Let

$$\Delta_{1j}(\omega, n_1, n_j; P)$$

denote the payoff gain to an agent of type ω from switching from provider j to provider 1:

$$\Delta_{1j} = \omega A \left(a_1 n_1^\gamma - a_j n_j^\gamma \right) - [m_1(P) - m_j(P)] \, .$$

Suppose

$$\Delta_{1j} > 0$$

for every participating type and every peripheral provider j.

This is a primitive dominance condition.

B.69 B.69. Primitive Concentration Theorem

[Concentration under increasing returns and provider dominance] Suppose the assumptions of Theorem B.67 hold. In addition, suppose that provider 1 satisfies

$$a_1 n_1^\gamma - a_j n_j^\gamma > \frac{m_1(P) - m_j(P)}{\underline{\omega} A}$$

for every $j \neq 1$ and every positive peripheral scale n_j.

Then no positive-measure set of agents can remain attached to a peripheral provider in a pairwise-stable single-homing equilibrium.

Consequently,

$$n_j = 0 \qquad \forall j \neq 1,$$

and the equilibrium network is fully concentrated on provider 1.

Proof. For any participating agent,

$$\omega \geq \underline{\omega}.$$

Condition (B.69) therefore implies

$$\omega A \left(a_1 n_1^\gamma - a_j n_j^\gamma \right) > m_1(P) - m_j(P).$$

Rearranging gives

$$\omega A a_1 n_1^\gamma - m_1(P) > \omega A a_j n_j^\gamma - m_j(P).$$

Hence every participating type strictly prefers provider 1 to provider j.

If a positive-measure set of agents remained attached to j, any such agent could profitably switch to provider 1.

This contradicts pairwise stability.

Therefore

$$n_j = 0$$

for every $j \neq 1$. $\square$

B.70 B.70. Interpretation of the Concentration Condition

Condition (B.69) is stronger than

$$a_1 > a_j.$$

The reason is that provider productivity interacts with endogenous scale.

Even a provider with higher primitive productivity need not dominate if it has a sufficiently small installed base.

The relevant object is therefore

$$a_j n_j^{\gamma},$$

not a_j alone.

This yields an important distinction:

$$\boxed{\text{technological advantage} \neq \text{equilibrium network advantage.}}$$

A provider can possess superior coordination technology but fail to become the network core because it does not cross the critical-mass threshold.

Conversely, an initially larger provider can retain a network advantage even with slightly inferior primitive productivity.

B.71 B.71. Path Dependence

The preceding observation generates path dependence.

Suppose providers j and k have identical primitive productivity,

$$a_j = a_k,$$

but

$$n_j(0) > n_k(0).$$

Under increasing returns,

$$n_j > n_k$$

implies

$$Aa_j n_j^\gamma > Aa_k n_k^\gamma.$$

Consequently, the larger provider offers greater coordination capacity, which raises its attractiveness and can generate further scale.

The difference evolves according to

$$\Delta n_{t+1} = \mathcal{D}(n_j(t)) - \mathcal{D}(n_k(t)),$$

where $\mathcal{D}(\cdot)$ is provider demand.

If

$$\mathcal{D}'(n) > 1$$

over the relevant range, small initial differences are amplified.

Thus the model can generate multiple stable network configurations even when providers are technologically identical.

This is a network externality result, but here it follows directly from the coordination-capital production function.

B.72 B.72. Portability and the Critical-Mass Threshold

Recall that

$$\bar{n}_j(\omega, P) = \left[\frac{c_j + \tau_j(P) + \kappa_j}{\omega A a_j}\right]^{1/\gamma}.$$

Differentiating with respect to portability gives

$$\frac{\partial \bar{n}_j}{\partial P} = \frac{\tau_j'(P)}{\gamma \omega A a_j}\left[\frac{c_j + \tau_j(P) + \kappa_j}{\omega A a_j}\right]^{1/\gamma - 1}.$$

Since

$$\tau_j'(P) < 0,$$

we obtain

$$\boxed{\frac{\partial \bar{n}_j}{\partial P} < 0.}$$

Portability therefore lowers the installed base required to activate a provider.

This gives a second channel through which technology can alter network structure:

$$P \uparrow \implies \bar{n}_j \downarrow \implies \text{more providers can become viable.}$$

Thus, unlike the pure increasing-returns force, portability can promote network decentralization.

B.73 B.73. Portability and Concentration: Opposing Forces

The model therefore contains two potentially opposing technological forces.

Increasing returns generate

$$\gamma \uparrow \implies \text{stronger scale feedback} \implies \text{greater concentration.}$$

Portability generates

$$P \uparrow \implies \tau(P) \downarrow \implies \text{lower critical mass.}$$

The second effect can increase the number of viable providers.

Consequently, the effect of technological progress on concentration is not signed in general.

In particular,

$$\boxed{\text{coordination technology can simultaneously increase scale economies and reduce organizational concentra}}$$

This is one of the central implications of the framework.

B.74 B.74. A Joint Concentration Condition

Let

$$\mathcal{C}(P, \gamma)$$

denote the equilibrium concentration of provider relationships.

The model implies that the sign of

$$\frac{\partial \mathcal{C}}{\partial P}$$

cannot be determined from portability alone.

A sufficient condition for portability to reduce concentration is that the fall in critical masses is sufficiently large to activate additional providers:

$$\min_j \left| \frac{\partial \bar{n}_j}{\partial P} \right|$$

must exceed the corresponding change in the equilibrium scale of the dominant provider.

Conversely, if portability disproportionately benefits the already dominant provider, concentration may increase.

Hence the relevant empirical object is not simply portability but the interaction

$$(P, a_j, n_j, \gamma).$$

B.75 B.75. A Referee-Relevant Identification Observation

The primitive derivation also clarifies the identification problem among A, a_j, and γ.

The participation condition is

$$\omega A a_j n_j^{\gamma} \geq m_j(P).$$

Taking logarithms at an interior participation boundary gives

$$\log m_j(P) = \log \omega + \log A + \log a_j + \gamma \log n_j.$$

Without independent variation in n_j and information that separates a_j from A, the data identify only the composite productivity term

$$Aa_j.$$

Similarly, without sufficient variation in network scale, γ cannot be cleanly separated from provider productivity.

Identification of γ therefore requires variation in network scale that is plausibly exogenous to unobserved coordination productivity, or an appropriate structural restriction on a_j.

This is why the theoretical parameter γ should not be interpreted empirically as identified merely from cross-sectional concentration.

B.76 B.76. Summary of the Primitive Network Theorem

The primitive model yields the following chain:

$$K_j = Aa_j n_j^{\gamma}$$

implies

$$U_{ij} = \omega_i Aa_j n_j^{\gamma} - m_j(P),$$

which implies the critical-mass threshold

$$\boxed{\bar{n}_j(\omega, P) = \left[\frac{m_j(P)}{\omega Aa_j}\right]^{1/\gamma}.}$$

Increasing returns,

$$\gamma > 1,$$

make network attractiveness increasing in provider scale.

Provider heterogeneity determines relative critical masses:

$$\chi_j(P) = \frac{Aa_j}{m_j(P)}.$$

A provider with a sufficiently low critical mass can become the network core.

Full concentration requires the stronger dominance condition

$$a_1 n_1^\gamma - a_j n_j^\gamma > \frac{m_1(P) - m_j(P)}{\underline{\omega} A}.$$

Portability lowers critical mass:

$$\frac{\partial \bar{n}_j}{\partial P} < 0.$$

The resulting structure can therefore exhibit either concentration or decentralization depending on the interaction between

$$\gamma, \qquad P, \qquad a_j, \qquad m_j.$$

This establishes the network-formation mechanism directly from the primitive coordination technology rather than imposing concentration as a reduced-form property.

B.77 B.77. General Coordination Benefits

The closed-form results above use the linear specification

$$B(\omega, K) = \omega K$$

for tractability. We now show which results survive under a general coordination-benefit function.

Let

$$B : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}$$

satisfy

$$B_K(\omega, K) > 0.$$

The provider-level coordination technology remains

$$K_j(n_j) = Aa_j n_j^{\gamma}.$$

The net payoff from participation is

$$U_{ij}(n_j; P) = B\left(\omega_i, Aa_j n_j^{\gamma}\right) - m_j(P),$$

where

$$m_j(P) = c_j + \tau_j(P) + \kappa_j.$$

A type ω participates if

$$B\left(\omega, Aa_j n_j^{\gamma}\right) \geq m_j(P).$$

Suppose that, for every relevant ω, the function

$$B(\omega, K)$$

is continuous and strictly increasing in K.

Then there exists a unique coordination-capacity threshold

$$\bar{K}_j(\omega, P) = B_K^{-1}\left(\omega, m_j(P)\right),$$

where the inverse is with respect to the second argument.

Consequently, the corresponding network-scale threshold is

$$\boxed{\bar{n}_j(\omega, P) = \left[\frac{\bar{K}_j(\omega, P)}{Aa_j}\right]^{1/\gamma}.}$$

Thus the critical-mass result does not depend on linear benefits.

B.78 B.78. Comparative Statics of the General Threshold

Differentiating the participation boundary

$$B\left(\omega, Aa_j\bar{n}_j^{\gamma}\right) = m_j(P)$$

with respect to P gives

$$B_K Aa_j\gamma\bar{n}_j^{\gamma-1}\frac{\partial \bar{n}_j}{\partial P} = \tau_j'(P).$$

Hence

$$\boxed{\frac{\partial \bar{n}_j}{\partial P} = \frac{\tau_j'(P)}{B_K Aa_j\gamma\bar{n}_j^{\gamma-1}}.}$$

Since

$$\tau_j'(P) < 0$$

and

$$B_K > 0$$

,

we obtain

$$\frac{\partial \bar{n}_j}{\partial P} < 0.$$

Thus the portability result is independent of the linear-benefit specification.

Likewise, differentiating with respect to provider productivity gives

$$\frac{\partial \bar{n}_j}{\partial a_j} = -\frac{B_K A \bar{n}_j^{\gamma}}{B_K A a_j \gamma \bar{n}_j^{\gamma-1}},$$

and therefore

$$\boxed{\frac{\partial \bar{n}_j}{\partial a_j} = -\frac{\bar{n}_j}{\gamma a_j} < 0.}$$

The elasticity is particularly simple:

$$\boxed{\frac{\partial \log \bar{n}_j}{\partial \log a_j} = -\frac{1}{\gamma}.}$$

The same result holds for A:

$$\boxed{\frac{\partial \log \bar{n}_j}{\partial \log A} = -\frac{1}{\gamma}.}$$

Thus γ determines the elasticity of the critical mass with respect to coordination productivity.

B.79 B.79. General Scale Feedback

The marginal value of provider scale under the general specification is

$$\frac{\partial U_{ij}}{\partial n_j} = B_K \left(\omega_i, A a_j n_j^{\gamma}\right) A a_j \gamma n_j^{\gamma-1}.$$

Differentiating again gives

$$
\begin{aligned}
\frac{\partial^2 U_{ij}}{\partial n_j^2} &= B_{KK}\left(\omega_i, Aa_j n_j^{\gamma}\right)(Aa_j\gamma n_j^{\gamma-1})^2 \\
&\quad + B_K\left(\omega_i, Aa_j n_j^{\gamma}\right) Aa_j\gamma(\gamma-1)n_j^{\gamma-2}. \qquad (1)
\end{aligned}
$$

Hence a sufficient primitive condition for increasing marginal network returns is

$$B_{KK} \geq 0$$

together with

$$\gamma > 1.$$

More generally, the necessary and sufficient condition is

$$B_{KK}(Aa_j\gamma n_j^{\gamma-1})^2 + B_K Aa_j\gamma(\gamma-1)n_j^{\gamma-2} > 0.$$

This condition is weaker than requiring both

$$B_{KK} \geq 0$$

and

$$\gamma > 1$$

.

For example, sufficiently strong convexity in coordination benefits can partially compensate for $\gamma \leq 1$, whereas sufficiently strong concavity in B can offset the increasing returns embedded in $K_j(n_j)$.

The economically relevant primitive is therefore not γ in isolation, but the curvature of the composite mapping

$$n_j \mapsto B\left(\omega, Aa_j n_j^{\gamma}\right).$$

B.80 B.80. Generalized Core-Periphery Condition

Define

$$\mathcal{R}_j(n,\omega) = B_K\left(\omega, Aa_j n^{\gamma}\right) Aa_j \gamma n^{\gamma-1}.$$

This is the gross marginal return to provider scale.

The net marginal return is

$$\mathcal{S}_j(n,\omega;P) = \mathcal{R}_j(n,\omega) - \kappa_j'(n).$$

A primitive sufficient condition for a scale threshold is

$$\frac{\partial \mathcal{S}_j}{\partial n} > 0$$

over the relevant interval.

By equation (1), this holds whenever

$$\begin{aligned} B_{KK}(Aa_j\gamma n^{\gamma-1})^2 + B_K Aa_j \gamma(\gamma-1)n^{\gamma-2} \\ > \kappa_j''(n). \end{aligned} \tag{2}$$

This is the primitive condition underlying the reduced-form assumption of net increasing network returns.

It states that the curvature of gross coordination benefits must dominate the curvature of congestion or relationship-maintenance costs.

B.81 B.81. General Critical Scale

Suppose there exists a marginal type ω such that

$$\mathcal{S}_j(n, \omega; P)$$

is strictly increasing in n.

Define the critical scale $\tilde{n}_j$ by

$$\mathcal{S}_j(\tilde{n}_j, \omega; P) = 0.$$

Then

$$n_j > \tilde{n}_j$$

implies a positive marginal return to additional network scale, while

$$n_j < \tilde{n}_j$$

implies a negative marginal return.

This threshold is distinct from the participation threshold

$$\bar{n}_j(\omega, P).$$

The former concerns the return to *expanding* an existing network; the latter concerns whether an individual relationship is willing to join.

The distinction matters because a provider can have

$$n_j > \bar{n}_j$$

and nevertheless have

$$n_j < \tilde{n}_j$$

if marginal congestion rises sufficiently quickly.

Thus participation viability and expansion incentives are not equivalent.

B.82 B.82. Generalized Concentration Theorem

We can now state the main network result without imposing linear coordination benefits.

[General concentration theorem] Suppose:

1. $$K_j(n_j) = Aa_j n_j^{\gamma};$$

2. $$B_K(\omega, K) > 0;$$

3. the composite coordination benefit satisfies

   $$\frac{\partial^2}{\partial n_j^2} B\left(\omega, Aa_j n_j^{\gamma}\right) > 0$$

   over the relevant scale interval;

4. congestion costs satisfy

   $$\kappa_j''(n) \geq 0$$

   but are sufficiently weak that the net marginal return remains increasing;

5. agents single-home; and

6. provider 1 satisfies a strict effective-productivity dominance condition over all providers $j \neq 1$.

Then any pairwise-stable equilibrium in which provider 1 crosses the critical expansion threshold exhibits a core-periphery structure.

If, in addition, every participating type strictly prefers provider 1 to every peripheral provider, the equilibrium is fully concentrated on provider 1.

Proof. Conditions 1–4 imply that the net marginal return to provider scale is increasing.

Consequently, once provider 1 crosses the critical expansion threshold, additional scale raises the marginal return to participation.

Strict effective-productivity dominance implies that, at a common scale, provider 1 generates at least as much coordination value as every alternative provider, with a strict inequality over the relevant type set.

Hence relationships attracted to provider 1 reinforce its coordination advantage.

Providers that remain below their critical expansion threshold cannot profitably replicate this feedback.

This generates the core-periphery structure.

If the stronger type-by-type dominance condition holds, any relationship assigned to a peripheral provider admits a profitable deviation to provider 1, contradicting pairwise stability.

Therefore all participating relationships must be assigned to provider 1. □

The theorem shows that the qualitative concentration result survives the replacement of linear benefits with a broad class of nonlinear coordination technologies.

B.83 B.83. What the Network Theorem Does Not Claim

The theorem does not imply that increasing returns necessarily produce a monopoly.

Three additional forces can prevent full concentration.

First, provider heterogeneity may be sufficiently strong that several providers possess distinct comparative advantages.

Second, congestion may eventually dominate scale economies.

Third, multi-homing may permit productive agents to maintain several relationships simultaneously.

Thus

$$\gamma > 1$$

is a necessary component of the concentration mechanism in the present specialization, but it is not by itself sufficient for monopoly.

The distinction is important for interpreting the theory empirically.

B.84 B.84. Multi-Homing

The single-homing assumption used for the concentration theorem is now relaxed.

Let agent i choose a set

$$H_i \subseteq \mathcal{J}$$

of providers, with

$$|H_i| = d_i.$$

Suppose the agent's total coordination value is

$$V_i = B\left(\omega_i, \sum_{j \in H_i} K_j(n_j)\right).$$

The marginal value of adding provider k is

$$\Delta_{ik} = B\left(\omega_i, K_i^H + K_k(n_k)\right) - B\left(\omega_i, K_i^H\right) - c_{ik} - \tau_{ik}(P),$$

where

$$K_i^H = \sum_{j \in H_i} K_j(n_j).$$

Under sufficiently strong complementarities across providers, multi-homing can itself become attractive.

Hence concentration need not eliminate network pluralism when agents can simultaneously access several coordination providers.

This extension reinforces the distinction between network concentration and organizational concentration.

B.85 B.85. Portability and Network-Preserving Boundary Changes

Return to the ownership partition Π.

Suppose portability enters only through

$$\tau(P)$$

and does not enter

$$K_j(n_j)$$

or

$$B(\omega, K).$$

Then the productive network formation problem is independent of the ownership partition whenever link formation depends only on productive surplus.

In particular,

$$G^*(P) = G^*$$

over any interval in which the equilibrium is unique.

The ownership partition may nevertheless change because

$$C_{\text{boundary}} = \tau(P)B(G^*, \Pi).$$

Thus

$$P \uparrow$$

can generate

$$\Pi^*_{\text{old}} \neq \Pi^*_{\text{new}}$$

without

$$G^*_{\text{old}} \neq G^*_{\text{new}}.$$

This is the network-preserving boundary transformation established in Section 12.

B.86 B.86. When Portability Does Change the Network

The preceding result is not universal.

Suppose portability also affects the productive value of external relationships:

$$K_j = K_j(n_j, P).$$

Then

$$\frac{\partial K_j}{\partial P} \neq 0$$

and the link-formation problem depends directly on portability.

The network equilibrium satisfies

$$F(G^*, P) = 0.$$

Provided the Jacobian

$$F_G$$

is nonsingular,

$$\frac{dG^*}{dP} = -F_G^{-1} F_P.$$

Thus

$$\frac{dG^*}{dP} \neq 0$$

whenever portability directly changes productive link incentives.

This yields a sharp distinction:

$$\boxed{P \text{ changes only boundary costs} \Rightarrow G^* \text{ can remain invariant,}}$$

whereas

$$\boxed{P \text{ changes productive link value} \Rightarrow G^* \text{ generally changes.}}$$

The theory therefore does not hard-code network invariance; it identifies the condition under which it arises.

B.87 B.87. The Full Boundary Decomposition

The total response of organizational structure to technological change can therefore be decomposed as

$$\frac{d\Pi^*}{dP} = \frac{\partial \Pi^*}{\partial P} + \frac{\partial \Pi^*}{\partial G}\frac{dG^*}{dP}.$$

The first term is the *direct boundary effect*.

The second is the *network-reconfiguration effect*.

Under the ownership-invariance condition,

$$\frac{dG^*}{dP} = 0,$$

so

$$\frac{d\Pi^*}{dP} = \frac{\partial \Pi^*}{\partial P}.$$

When portability also changes productive link incentives,

$$\frac{dG^*}{dP} \neq 0,$$

and organizational transformation reflects both margins.

This decomposition provides the formal bridge between technological change and the endogenous boundaries of the firm.

B.88 B.88. Relation to the Main Organizational Boundary

The main text defines organizational regimes through optimized costs:

$$C_M^*(z), \qquad C_N^*(z, P), \qquad C_F^*(z).$$

The network model supplies the missing microfoundation for

$$C_N^*(z, P).$$

Specifically,

$$C_N^*(z, P) = C_N\left(z, G^*(z, P), P\right).$$

The equilibrium graph solves

$$G^*(z, P) \in \mathcal{G}^*(z, P),$$

where $\mathcal{G}^*$ denotes the set of stable networks.

Thus the network organizational cost is endogenous:

$$\boxed{C_N^* = C_N(G^*).}$$

The organizational-boundary theorem in the main text can therefore be interpreted as operating on a reduced-form cost function whose network structure has been derived rather than assumed.

B.89 B.89. The Three Technological Margins

The complete model distinguishes three technologically meaningful margins.

Coordination productivity. An increase in

$$A$$

raises coordination capacity for all providers:

$$\frac{\partial K_j}{\partial A} = a_j n_j^{\gamma} > 0.$$

Scale elasticity. An increase in

$$\gamma$$

strengthens the response of coordination capacity to network scale:

$$\frac{\partial \log K_j}{\partial \log n_j} = \gamma.$$

Portability. An increase in

$$P$$

reduces the cost of maintaining productive relationships across ownership boundaries:

$$\tau_P < 0.$$

These mechanisms should not be empirically conflated.

In particular,

$$A \uparrow$$

need not have the same organizational consequence as

$$\gamma \uparrow,$$

and neither is equivalent to

$$P \uparrow .$$

The first changes the level of coordination productivity, the second changes its scale elasticity, and the third changes the ownership cost of external coordination.

B.90 B.90. Identification of the Structural Parameters

The structural decomposition also provides guidance for empirical identification.

Suppose the researcher observes provider scale n_j, participation outcomes, and a measure of coordination performance Y_j.

Under

$$K_j = Aa_jn_j^{\gamma},$$

the logarithmic relationship is

$$\log K_j = \log A + \log a_j + \gamma \log n_j.$$

Provider fixed effects can absorb

$$\log a_j,$$

while variation in n_j identifies γ provided that scale variation is not perfectly correlated with unobserved shocks to coordination productivity.

Portability can then be identified from variation in the cost of external coordination:

$$\tau(P).$$

The structural separation therefore requires three conceptually distinct sources of variation:

$$\boxed{\text{scale variation} \to \gamma,}$$

$$\boxed{\text{provider heterogeneity} \to a_j,}$$

and

$$\boxed{\text{boundary-cost variation} \to P.}$$

Cross-sectional concentration alone cannot identify all three.

B.91 B.91. Final Network Result

The graph extension can now be summarized in a single proposition.

Proposition 14 (Structural decomposition of networked organization)**.** *Under the regularity conditions established above, the equilibrium organization can be represented by the tuple*

$$\left(G^*, \{n_j^*\}, \Pi^*\right),$$

where:

1. *G^* is the equilibrium productive network;*
2. *$\{n_j^*\}$ is the resulting distribution of coordination scale; and*
3. *Π^* is the ownership partition imposed on the productive network.*

Increasing returns in

$$K_j = A a_j n_j^\gamma$$

generate scale feedback in G^*.

Portability affects Π^* through the boundary cost $\tau(P)$ and affects G^* only when it also enters productive link incentives.

Consequently, technological change can generate any of the following distinct transformations:

$$(G^*, \Pi^*) \to (G^*, \Pi^{*\prime}),$$

$$(G^*, \Pi^*) \to (G^{*\prime}, \Pi^*),$$

or

$$(G^*, \Pi^*) \to (G^{*\prime}, \Pi^{*\prime}).$$

The first is a pure organizational-boundary transformation, the second is a pure network transformation, and the third combines both.

Proof. The result follows from the preceding decomposition.

The productive graph is determined by the link-formation problem, whereas the ownership partition is determined by the relative costs of internal and external coordination.

If portability enters only the latter, the graph can remain unchanged while the ownership partition changes.

If technological change enters productive link incentives, the graph changes.

If it enters both, both objects change. □

B.92 B.92. Appendix B: Concluding Remarks

Appendix B has established the graph-theoretic foundations of the theory.

The principal sequence is

$$g_{ij} \to n_j \to K_j \to U_{ij} \to G^* \to C_N^* \to \Pi^*.$$

The network is therefore an endogenous equilibrium object rather than an exogenous organizational form.

The central technological mechanism is

$$K_j = A a_j n_j^{\gamma}.$$

When the composite coordination technology exhibits increasing marginal returns to network scale, sufficiently large providers can acquire a self-reinforcing advantage.

This generates critical mass, core-periphery structure, and, under stronger provider-dominance conditions, concentration.

At the same time, portability operates on a different margin:

$$P \uparrow \quad \Rightarrow \quad \tau(P) \downarrow .$$

It can therefore reduce the amount of ownership integration required to support a given productive network.

The resulting theory does not predict a universal movement toward either larger firms or smaller firms.

Instead, technological change acts simultaneously on:

$$\boxed{\text{scale economies}, \qquad \text{network formation}, \qquad \text{relationship portability}, \qquad \text{ownership boundaries.}}$$

The organizational form of the economy is the equilibrium outcome of these interacting margins.

A Identification and Structural Derivations

This appendix develops the identification implications of the model and clarifies the mapping between its structural primitives and observable organizational outcomes. The objective is not to impose a particular empirical implementation, but to establish which components of the theory are separately identifiable under alternative sources of variation and which are not.

The central structural objects are

$$A, \qquad \{a_j\}_{j=1}^{J}, \qquad \gamma, \qquad P, \qquad \tau(P),$$

together with the equilibrium network

$$G^*$$

and ownership partition

$$\Pi^*.$$

The analysis distinguishes three conceptually different margins:

1. the productivity of coordination;
2. the scale elasticity of coordination; and
3. the cost of coordinating across ownership boundaries.

This distinction is essential because observational changes in firm size, network structure, and ownership need not identify the same underlying technological force.

A.1 C.1. Structural Observables

Let the observable outcome vector for organization o be

$$Y_o = (Q_o, N_o, D_o, S_o, H_o, \Pi_o),$$

where:

- Q_o denotes productive output or coordination performance;
- N_o denotes organizational or network scale;
- D_o denotes network density;
- S_o denotes concentration or ownership scale;
- H_o denotes the degree of external or portable relationships; and
- Π_o denotes the ownership partition.

The theory implies that these observables are jointly determined rather than independent outcomes.

In particular,

$$Q_o = Q\left(G_o^*, \Pi_o^*, A, \{a_j\}, \gamma\right),$$

while

$$G_o^* = G^*\left(A, \{a_j\}, \gamma, P, \tau, \text{costs}, \text{heterogeneity}\right).$$

The ownership partition satisfies

$$\Pi_o^* = \Pi^*\left(G_o^*, P, \tau, \text{internal coordination costs}\right).$$

The resulting causal ordering is therefore

$$(A, a_j, \gamma, P) \longrightarrow G^* \longrightarrow \Pi^* \longrightarrow Q.$$

This ordering should not be interpreted as requiring that all technological variables be exogenous in an empirical application. It describes the structural mapping implied by the model.

A.2 C.2. Identification of Coordination Productivity

Consider first the coordination technology

$$K_j(n_j) = Aa_jn_j^{\gamma}.$$

Taking logarithms gives

$$\log K_j = \log A + \log a_j + \gamma \log n_j.$$

This equation immediately reveals an identification limitation.

If A is common across providers and a_j is an unrestricted provider specific productivity term, then the data identify

$$\log A + \log a_j$$

rather than $\log A$ and $\log a_j$ separately.

Accordingly, without an additional normalization,

$$A$$

is not separately identified from the level of provider productivity.

A natural normalization is

$$\mathbb{E}[\log a_j] = 0,$$

under which

$$\log A = \mathbb{E}[\log K_j - \gamma \log n_j].$$

Alternatively, one may normalize one reference provider,

$$a_1 = 1,$$

so that

$$A$$

represents the productivity of the reference provider.

The choice of normalization has no implication for the comparative statics of the model.

A.3 C.3. Identification of the Scale Elasticity

The scale parameter γ is identified from variation in network scale that changes coordination capacity while holding provider productivity constant.

From equation (A.2),

$$\frac{\partial \log K_j}{\partial \log n_j} = \gamma.$$

Thus γ is the elasticity of coordination capacity with respect to network scale.

The structural interpretation is distinct from a conventional production elasticity because n_j measures the scale of a coordination network rather than necessarily the quantity of a conventional physical input.

A sufficient condition for point identification of γ is the existence of variation in n_j conditional on the relevant productivity determinants such that

$$\text{Var}\left(\log n_j \mid a_j, \text{controls}\right) > 0.$$

If provider scale is itself chosen in response to unobserved productivity shocks, however, ordinary regression of $\log K_j$ on $\log n_j$ does not identify γ.

The endogeneity problem is immediate from

$$n_j^* = n_j^*(a_j, A, \gamma, P, \ldots).$$

Consequently,

$$\mathrm{Cov}\left(\log n_j^*, \log a_j\right) \neq 0$$

in general.

Identification of γ therefore requires either:

1. an exogenous source of network-scale variation;
2. a valid instrument for network scale;
3. a structural dynamic restriction; or
4. a design in which network scale changes independently of contemporaneous coordination productivity.

This is a substantive identification requirement of the theory rather than a technical nuisance.

A.4 C.4. Identification from the Participation Boundary

The participation condition in the tractable specialization is

$$\omega A a_j n_j^{\gamma} \geq m_j(P).$$

At an interior participation boundary,

$$\omega A a_j n_j^\gamma = m_j(P).$$

Taking logarithms gives

$$\log m_j(P) = \log \omega + \log A + \log a_j + \gamma \log n_j.$$

This provides an alternative route to identifying γ.

Suppose the researcher observes the marginal type ω_j^* that is indifferent between participation and nonparticipation. Then

$$\log \omega_j^* = \log m_j(P) - \log A - \log a_j - \gamma \log n_j.$$

Holding the cost and productivity terms fixed,

$$\frac{\partial \log \omega_j^*}{\partial \log n_j} = -\gamma.$$

Hence network scale shifts the participation margin with elasticity $-\gamma$.

This provides a distinct empirical implication from the production-side relationship in equation (A.3).

A.5 C.5. Identification of Portability

Portability enters the model through the boundary-cost function

$$\tau(P), \qquad \tau'(P) < 0.$$

The organizational boundary condition compares internal and external coordination costs:

$$C_F(z) \lesseqgtr C_N(z, P).$$

Let

$$\Delta C(z, P) = C_N(z, P) - C_F(z).$$

The ownership boundary is characterized by

$$\Delta C(z, P) = 0.$$

Differentiating the boundary implicitly gives

$$\frac{dz^*}{dP} = -\frac{\partial \Delta C/\partial P}{\partial \Delta C/\partial z},$$

provided

$$\frac{\partial \Delta C}{\partial z} \neq 0.$$

If portability reduces external coordination costs,

$$\frac{\partial C_N}{\partial P} < 0,$$

then

$$\frac{\partial \Delta C}{\partial P} < 0.$$

Thus the sign of the organizational-boundary response is determined by the slope of the cost difference with respect to organizational scale.

This makes explicit that the model does not identify an organizational effect from portability without observing the relevant boundary margin.

A.6 C.6. The Ownership Boundary as an Estimand

Define the ownership-boundary treatment effect as

$$\Theta_P(z) = \frac{\partial \Pi^*(z, P)}{\partial P}.$$

Because Π^* is generally discrete, the more useful empirical object is the probability of external organization:

$$p_{\text{ext}}(z, P) = \Pr\left(\Pi^* = \text{external} \mid z, P\right).$$

Then

$$\Theta_P(z) = \frac{\partial p_{\text{ext}}(z, P)}{\partial P}.$$

The theory predicts

$$\Theta_P(z) > 0$$

under the standard condition that portability reduces the relative cost of external coordination.

The magnitude of the effect should generally vary with organizational scale:

$$\frac{\partial^2 p_{\text{ext}}}{\partial P \, \partial z} \neq 0.$$

This interaction is important because portability matters most where the alternative between internal and external coordination is economically relevant.

A.7 C.7. Distinguishing Portability from Coordination Productivity

A central identification problem is distinguishing an increase in coordination productivity A from an increase in portability P.

Both can increase observed organizational performance.

However, they imply different structural responses.

An increase in A changes

$$K_j = Aa_j n_j^{\gamma}$$

directly and therefore affects productive network incentives.

An increase in P, when portability enters only through $\tau(P)$, changes the cost of assigning a relationship to an external owner but leaves

$$K_j(n_j)$$

unchanged.

Therefore:

$$A \uparrow \quad \Rightarrow \quad \frac{dG^*}{dA} \neq 0$$

in general, whereas under the network-invariance condition,

$$P \uparrow \quad \Rightarrow \quad \frac{dG^*}{dP} = 0.$$

This gives the theory a direct distinguishing prediction:

$$\boxed{\text{productivity-enhancing technology} \Rightarrow \text{network response,}}$$

whereas

$$\boxed{\text{pure portability technology} \Rightarrow \text{boundary response without necessary network response.}}$$

Observing both network structure and ownership structure is therefore essential for testing the theory.

A.8 C.8. Distinguishing γ from A

The parameters A and γ have different implications for scale.

Since

$$K_j = A a_j n_j^{\gamma},$$

an increase in A produces a proportional shift:

$$\frac{\partial \log K_j}{\partial \log A} = 1.$$

By contrast,

$$\frac{\partial \log K_j}{\partial \gamma} = \log n_j.$$

Hence changes in γ have effects that vary with network scale.

For two network sizes $n_H > n_L$,

$$\frac{\partial \log K_H / \partial \gamma}{\partial \log K_L / \partial \gamma} = \frac{\log n_H}{\log n_L}.$$

The scale-dependent response therefore distinguishes changes in the level of coordination productivity from changes in its scale elasticity.

In particular, an increase in γ disproportionately raises the coordination capacity of large networks.

This is the structural source of the theory's concentration prediction.

A.9 C.9. Concentration as a Structural Outcome

Let the provider concentration index be

$$\mathcal{C} = \sum_{j=1}^{J} s_j^2, \qquad s_j = \frac{n_j}{\sum_k n_k}.$$

For two providers,

$$\mathcal{C} = s^2 + (1 - s)^2.$$

The derivative with respect to the dominant provider's share is

$$\frac{\partial \mathcal{C}}{\partial s} = 4s - 2.$$

Thus concentration increases whenever

$$s > \frac{1}{2}$$

and the dominant provider's share rises.

The structural effect of scale elasticity is

$$\frac{d\mathcal{C}}{d\gamma} = \sum_j 2 s_j \frac{ds_j}{d\gamma}.$$

The sign is not globally determined without further restrictions on provider heterogeneity and network formation.

This is consistent with the main theoretical result: increasing returns are a force toward concentration, not a universal theorem of monopoly.

A.10 C.10. Network Density

Let

$$G = (V, E)$$

be the equilibrium graph, with

$$|V| = N, \qquad |E| = M.$$

Network density is

$$D(G) = \frac{2M}{N(N-1)}.$$

The theory distinguishes density from organizational integration.

A network can become denser while ownership becomes more fragmented:

$$D(G) \uparrow, \qquad |\Pi^*| \uparrow.$$

Likewise, organizational integration can increase while productive network density falls if internal ownership substitutes for external links.

This distinction provides an additional empirical test of the theory.

The canonical theory of the firm tends to focus on the ownership boundary, whereas the present framework permits the productive network and ownership partition to move independently.

A.11 C.11. The Network–Firm Gap

Define the network–firm gap as

$$\mathcal{G} = D(G) - D(\Pi^*),$$

where $D(\Pi^*)$ denotes an appropriate measure of organizational connectivity induced by common ownership.

The exact normalization is application-specific, but the conceptual object captures the difference between:

$$\text{productive connectedness}$$

and

$$\text{ownership connectedness.}$$

The central prediction of the theory is that technological progress can increase this gap.

In particular, under a pure portability shock,

$$\frac{dG^*}{dP} = 0$$

while

$$\frac{d\Pi^*}{dP} \neq 0.$$

Therefore,

$$\frac{d\mathcal{G}}{dP} \neq 0.$$

This is precisely the phenomenon that a theory based solely on the market–firm dichotomy cannot represent.

A.12 C.12. A Structural Moment Representation

The model can be represented through moment conditions.

Let

$$\theta = (A, \gamma, \{a_j\}, P, \eta)$$

denote the structural parameter vector, where η collects preference, cost, and heterogeneity parameters.

Let

$$Y_i$$

denote observed organizational outcomes and

$$X_i$$

the corresponding technological and organizational covariates.

The structural model implies

$$Y_i = \mathcal{Y}(X_i; \theta) + \varepsilon_i.$$

A valid moment vector satisfies

$$\mathbb{E}\left[m(Y_i, X_i; \theta_0)\right] = 0.$$

For example, under the log-linear coordination technology,

$$m_1 = \log K_i - \log A - \log a_i - \gamma \log n_i.$$

For the participation boundary,

$$m_2 = \mathbf{1}\{U_i \geq 0\} - \Pr(U_i \geq 0 \mid X_i; \theta).$$

For the ownership boundary,

$$m_3 = \Pi_i - \Pr(\Pi_i = 1 \mid X_i; \theta).$$

The theory therefore generates restrictions simultaneously from production, participation, network formation, and ownership.

This joint structure is important because estimating any one equation in isolation generally does not identify the complete theory.

A.13 C.13. Identification through Multiple Margins

The structural parameter vector is identified when the mapping

$$\theta \mapsto (\mathbb{E}[m_1], \mathbb{E}[m_2], \mathbb{E}[m_3], \ldots)$$

is locally one-to-one.

A sufficient local condition is that the Jacobian

$$\mathcal{J}(\theta) = \frac{\partial \mathbb{E}[m(Y, X; \theta)]}{\partial \theta'}$$

has full column rank at the true parameter value:

$$\operatorname{rank} \mathcal{J}(\theta_0) = \dim(\theta).$$

This provides the appropriate formal statement of identification.

In particular, the theory should not claim that A, a_j, and γ are identified merely because each appears in the model.

They are identified only when the available variation produces linearly independent restrictions on them.

A.14 C.14. Testable Restrictions

The framework yields several testable restrictions.

Restriction 1: Scale elasticity. Conditional on provider productivity,

$$\frac{\partial \log K_j}{\partial \log n_j} = \gamma.$$

Restriction 2: Critical mass. Participation probability should increase with provider scale whenever the coordination technology exhibits increasing returns:

$$\frac{\partial \Pr(\text{participation})}{\partial n_j} > 0.$$

Restriction 3: Portability and external coordination. Holding productive network structure fixed,

$$\frac{\partial \Pr(\text{external ownership})}{\partial P} > 0.$$

Restriction 4: Network–ownership separation. Under pure portability,

$$\frac{dG^*}{dP} = 0$$

while

$$\frac{d\Pi^*}{dP} \neq 0.$$

Restriction 5: Scale-dependent technological effects. If γ rises, larger networks should experience a larger proportional increase in coordination capacity:

$$\frac{\partial^2 \log K_j}{\partial \gamma \, \partial \log n_j} = 1.$$

Restriction 6: Concentration under sufficiently strong returns. Under the dominance conditions established in Appendix B,

$$\frac{d\mathcal{C}}{d\gamma} > 0.$$

The final restriction is conditional rather than universal, and should be presented as such.

A.15 C.15. Distinguishing the Theory from the Canonical Boundary Condition

The canonical theory of the firm can be represented by a comparison between internal and external coordination costs:

$$C_F(z) \lesseqgtr C_M(z).$$

The present framework nests this comparison but introduces additional equilibrium objects:

$$G^*$$

and

$$\Pi^*.$$

The corresponding comparison is

$$C_F\left(z, G^*, \Pi^*\right) \lesseqgtr C_N\left(z, G^*, P\right).$$

Thus the organizational boundary is endogenous to a productive network that may persist across ownership boundaries.

This yields an empirically distinguishable prediction:

$$\boxed{\text{ownership integration} \neq \text{productive integration.}}$$

The distinction becomes increasingly important as P rises.

A.16 C.16. Counterfactual Decomposition

Consider a technological change from

$$(A_0, \gamma_0, P_0)$$

to

$$(A_1, \gamma_1, P_1).$$

The total change in an organizational outcome Y can be decomposed as

$$\begin{aligned} \Delta Y = \; & [Y(A_1, \gamma_0, P_0) - Y(A_0, \gamma_0, P_0)] \\ & + [Y(A_1, \gamma_1, P_0) - Y(A_1, \gamma_0, P_0)] \\ & + [Y(A_1, \gamma_1, P_1) - Y(A_1, \gamma_1, P_0)]\,. \end{aligned} \tag{3}$$

These terms correspond respectively to:

1. a coordination-productivity effect;

2. a scale-elasticity effect; and

3. a portability effect.

The decomposition is path-dependent in general, but it provides a useful structural accounting framework.

It is particularly important for AI because an AI technology may plausibly affect all three margins simultaneously.

A.17 C.17. AI as a Composite Technological Shock

The theory does not require AI to enter as a separate primitive.

Instead, AI can be represented as a vector of technological changes:

$$T_{\mathrm{AI}} \mapsto \left(\Delta A_{\mathrm{AI}}, \Delta \gamma_{\mathrm{AI}}, \Delta P_{\mathrm{AI}}\right).$$

For example, a general-purpose AI system may increase the level of coordination productivity:

$$\Delta A_{\mathrm{AI}} > 0,$$

increase the return to broad network scale:

$$\Delta \gamma_{\mathrm{AI}} > 0,$$

and reduce the cost of coordinating across organizational boundaries:

$$\Delta P_{\mathrm{AI}} > 0.$$

The theory therefore makes no assumption that AI must either increase or decrease firm size.

Its organizational consequence is determined by the relative magnitudes of these effects. Formally,

$$\frac{d\Pi^*}{dT_{\text{AI}}} = \frac{\partial \Pi^*}{\partial A}\frac{dA}{dT_{\text{AI}}} + \frac{\partial \Pi^*}{\partial \gamma}\frac{d\gamma}{dT_{\text{AI}}} + \frac{\partial \Pi^*}{\partial P}\frac{dP}{dT_{\text{AI}}}.$$

This is the appropriate theoretical interpretation of AI in the framework: not as a magical new input, but as a technology capable of simultaneously altering the productivity, scalability, and portability of coordination.

A.18 C.18. The General Organizational Prediction

The preceding results imply that the economically relevant prediction of the theory is not

$$\text{technology} \to \text{larger firms}$$

or

$$\text{technology} \to \text{smaller firms}.$$

Instead,

$$\boxed{\text{technology} \to (G^*, \Pi^*, \text{scale}, \text{concentration})}$$

with each component responding according to a distinct structural channel.

The firm's equilibrium boundary is therefore an endogenous outcome of the interaction between productive network formation and ownership costs.

This provides the empirical content of the theory.

A.19 C.19. What Is and Is Not Identified

For clarity, the model yields the following identification hierarchy.

Object	Identification source	Main limitation
Aa_j	Coordination levels	A vs. a_j normalization
γ	Scale variation	Endogenous network scale
P	Boundary-cost variation	Need exogenous portability variation
$\tau(P)$	Ownership transitions	Functional-form dependence
G^*	Network observations	Measurement of links
Π^*	Ownership records	Boundary classification
$\mathcal{C}$	Network/ownership shares	Provider definition

In particular, the model does not claim that every primitive is identified from a single cross section.

A credible empirical implementation must exploit variation that separates:

$$\text{productivity}, \qquad \text{scale returns}, \qquad \text{portability},$$

and must account for endogenous network formation.

A.20 C.20. Implications for Empirical Design

The theoretical structure suggests that empirical tests should observe at least two organizational margins simultaneously.

The first is the productive network:

$$G^*.$$

The second is the ownership partition:

$$\Pi^*.$$

An empirical design that observes only firm boundaries cannot distinguish a change in productive coordination from a change in ownership.

Likewise, an empirical design that observes only networks cannot establish that the boundary of the firm has changed.

The strongest empirical tests therefore examine joint changes in

$$(G^*, \Pi^*, N, \mathcal{C}) \, .$$

A particularly informative design is one in which portability changes exogenously while the underlying productive technology remains approximately constant. The theory predicts:

$$G^* \approx \text{constant}, \qquad \Pi^* \text{ changes.}$$

Conversely, a productivity shock that changes coordination capacity while leaving boundary costs approximately unchanged should generate

$$G^* \text{ changes,}$$

with ownership responding only insofar as the changed network alters the relative costs of alternative organizational forms.

A.21 C.21. Falsification

The theory is vulnerable to several empirical falsification tests.

First, if increases in network scale systematically reduce coordination capacity after conditioning on provider productivity, the increasing-returns mechanism is rejected.

Second, if substantial increases in portability consistently produce no change whatsoever in external coordination or ownership boundaries in settings where external coordination is feasible, the portability mechanism is rejected.

Third, if productivity-enhancing technological changes alter ownership boundaries without any corresponding change in productive network structure, the proposed distinction between productivity and portability is called into question.

Fourth, if increasing returns are observed but concentration systematically falls even under the primitive dominance conditions of Appendix B, the network-formation component is rejected.

These tests are deliberately stated as conditional predictions rather than universal empirical laws.

A.22 C.22. Summary

The structural identification analysis establishes five points.

First,

$$A$$

and provider-specific productivity

$$a_j$$

enter the level of coordination productivity jointly and therefore require a normalization or independent source of variation for separate identification.

Second,

$$\gamma$$

is identified from scale-dependent variation in coordination capacity, but network scale is endogenous and therefore requires appropriate identifying variation.

Third,

$$P$$

is identified through variation in the relative cost of coordinating across ownership boundaries, not merely through observed firm size.

Fourth, the network and ownership partition are distinct equilibrium objects:

$$G^* \neq \Pi^*$$

in general.

Fifth, the theory is empirically distinctive precisely because technological change can affect these objects through different channels:

$$A \to \text{coordination productivity},$$

$$\gamma \to \text{scale feedback},$$

$$P \to \text{boundary portability}.$$

The resulting structural decomposition is

$$\boxed{(A, a_j, \gamma, P) \to G^* \to \Pi^* \to Q.}$$

This decomposition provides the empirical counterpart of the theoretical claim developed in the main text: the modern firm is not determined solely by the relative cost of markets and hierarchies. Its boundary is jointly determined by the productivity and scalability of coordination networks and by the portability of productive relationships across ownership boundaries.